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1. For the curve given by $\mathbf{r}(t) = \frac{1}{3}t^3\mathbf{i} + \frac{1}{2}t^2\mathbf{j} + 2t\mathbf{k}$, find the following at the point $(0, 0, 0)$:

like ch 14 rev. #11

(a) the unit tangent vector

6 pts

$$\underline{T}(t) = \frac{\underline{r}'(t)}{\|\underline{r}'(t)\|} = \frac{\langle t^2, t, 2\rangle}{\sqrt{t^4+t^2+4}}$$

$$\underline{T}(0) = \frac{\langle 0,0,2\rangle}{2} = \langle 0,0,1\rangle$$

(b) the unit normal vector $\underline{N} = \dfrac{\underline{T}'(t)}{\|\underline{T}'(t)\|}$

6 pts

$$\underline{T}'(t) = \frac{\sqrt{t^4+t^2+4}\,\langle 2t,1,0\rangle - \frac{1}{2}\langle t^2,t,2\rangle \frac{4t^3+2t}{\sqrt{t^4+t^2+4}}}{t^4+t^2+4}$$

$$\underline{T}'(0) = \frac{2\langle 0,1,0\rangle - \frac{1}{2}\langle 0,0,1\rangle \cdot 0}{4} = \frac{\langle 0,2,0\rangle}{4}$$

$$\underline{N}(0) = \frac{\langle 0,\frac{1}{2},0\rangle}{\|\langle 0,\frac{1}{2},0\rangle\|} = \langle 0,1,0\rangle = \underline{N}(0)$$

(c) the curvature

6 pts

(i) $\underline{r}' = \langle t^2, t, 2\rangle$, $\underline{r}'' = \langle 2t, 1, 0\rangle$

$$K(0) = \frac{\left\| \begin{vmatrix} i & j & k \\ 0 & 0 & 2 \\ 0 & 1 & 0 \end{vmatrix} \right\|}{\|\langle 0,0,2\rangle\|^3}$$

$$= \frac{\|\langle -2,0,0\rangle\|}{2^3} = \frac{2}{8} = \frac{1}{4}$$

(ii) or simply $K(0) = \dfrac{\|\underline{T}'(0)\|}{\|\underline{r}'(0)\|} = \dfrac{\frac{1}{2}}{2} = \dfrac{1}{4}$

(OVER)

14.4, 15 HW

2. Find the velocity and position vectors of a particle that has the given acceleration and initial velocity and position: $\mathbf{a}(t) = \mathbf{k}$, $\mathbf{v}(0) = \mathbf{i} - \mathbf{j}$, $\mathbf{r}(0) = \mathbf{0}$. Put boxes around your answers.

10 pts

$\underline{v}(t) = t\underline{k} + \underline{C}$

$\underline{v}(0) = \underline{C} = \underline{i} - \underline{j} \Rightarrow \underline{v}(t) = t\underline{k} + \underline{i} - \underline{j}$

$\boxed{\underline{v}(t) = \underline{i} - \underline{j} + t\underline{k}}$ ← velocity

$\underline{r}(t) = t\underline{i} - t\underline{j} + \frac{t^2}{2}\underline{k} + \underline{K}$

position

$\underline{r}(0) = \underline{K} = \underline{0} \Rightarrow \boxed{\underline{r}(t) = t\underline{i} - t\underline{j} + \frac{t^2}{2}\underline{k}}$

15.4, 11 HW

3. Find the linearization of the function $f(x, y) = x\sqrt{y}$ at the point $(1, 4)$.

10 pts

$f_x(x,y) = \sqrt{y} \rightarrow f_x(1,4) = 2.$

$f_y(x,y) = \frac{1}{2} x y^{-1/2} \rightarrow f_y(1,4) = \frac{1}{2}\frac{1}{\sqrt{4}}$

$f(1,4) + f_x(1,4)(x-1) + f_y(1,4)(y-4)$

$= \boxed{2 + 2(x-1) + \frac{1}{4}(y-4)}$

$= 2x + \frac{1}{4}y - 1.$

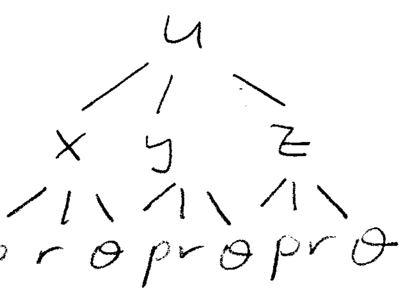
15.5, 25 HW

4. Let $u = x^2 + yz$, $x = pr\cos\theta$, $y = pr\sin\theta$, $z = p + r$. Find $\partial u/\partial p$ when $p = 2$, $r = 3$, $\theta = 0$.

10 pts

$\frac{\partial u}{\partial p} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial p} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial p} + \frac{\partial u}{\partial z}\frac{\partial z}{\partial p}.$

$= 2x(r\cos\theta) + z(r\sin\theta) + y(1)$



$r = 3, \theta = 0, x = 2 \cdot 3 \cdot \cos 0 = 6$

$y = 2 \cdot 3 \sin 0 = 0$

$z = 2 + 3 = 5.$

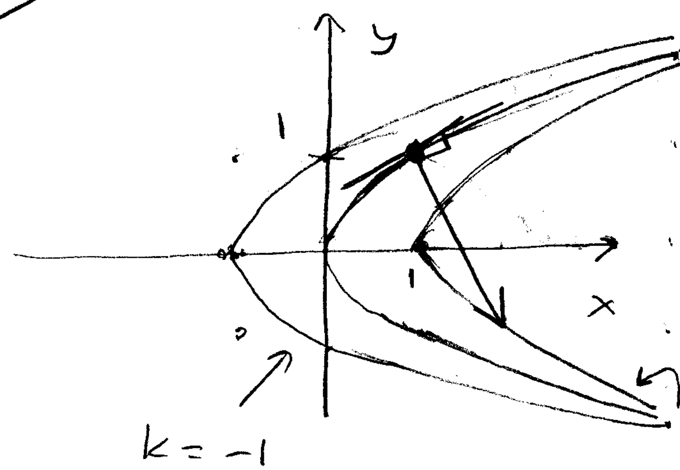
$\frac{\partial u}{\partial p} = 2(6)(3)(1) + 5(0) + 0(1) = \underline{36}$

5. Consider the function $f(x, y) = x - y^2. = k$

15.1, HW43

(a) Draw a contour map of the function showing several level curves.

10 pts



k=0

k=1.

$x - y^2 = 0 \rightarrow y^2 = x$

$x - y^2 = 1 \rightarrow y^2 = x - 1$

$x - y^2 = -1 \rightarrow y^2 = x + 1$

like 15.6, 21-26

(b) Find the direction (vector) of maximum rate of change of f at the point $(1, 1)$. Draw the vector with its tail at $(1, 1)$ on your sketch in Part (a). ✓

10 pts

$$\nabla f(1,1) = \langle 1, -2y \rangle \Big|_{\substack{x=1 \\ y=1}}$$

$$= \langle 1, -2 \rangle$$

on-line asst 4 #1.

6. Find the limit $\displaystyle\lim_{(x,y)\to(0,0)} \frac{(x+5y)^2}{x^2+25y^2}$ if it exists, or show why it does not exist. Explain your answer in one complete sentence.

Let $y = mx$

10 pts

$$\lim_{x \to 0} \frac{(x + 5mx)^2}{(x^2 + 25m^2x^2)} = \lim_{x\to 0} \frac{x^2 + 10mx^2 + 25m^2x^2}{x^2 + 25m^2x^2}$$

$$= \lim_{x \to 0} \frac{\cancel{x^2}(1 + 10m + 25m^2)}{\cancel{x^2}(1 + 25m^2)} \leftarrow \text{different values for different } m\text{'s.}$$

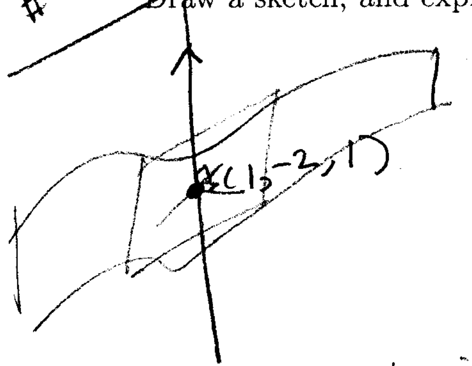
The fnc aproaches different values as $(x, y) \to (0,0)$ along different lines, so the limit does not exist.

(OVER)

ch15 Rev #25(b)

Find the equations of the normal line to the surface $z = 3x^2 - y^2 + 2x$ at the point $(1, -2, 1)$. Draw a sketch, and explain your work with sentences for purposes of partial credit.

10 pts



$$\underbrace{3x^2 - y^2 + 2x - z}_{F(x,y,z)} = 0.$$

$$\nabla F = \langle 6x+2, -2y, -1 \rangle.$$

$$\nabla F(1,-2,1) = \langle 8, 4, -1 \rangle.$$

$$\boxed{\begin{aligned} x &= 1 + 8t \\ y &= -2 + 4t \\ z &= 1 + (-1)t. \end{aligned}}$$

The normal line is orthogonal to the surface. The direction vector is $\nabla F(1,-2,1)$. The line passes through $P(1,-2,1)$.

15,7 #5

8. Find the local maximum, minimum, and saddle point(s) of the function $f(x, y) = 9 - 2x + 4y - x^2 - 4y^2$. Put a box around your answer, clearly labeling point(s) as max/min/saddle.

12 pts

$f_x = -2 - 2x = 0 \rightarrow x = -1$
$f_y = 4 - 8y = 0 \rightarrow y = \frac{1}{2}$ } critical point.

$f_{xx} = -2$
$f_{yy} = -8$
$f_{xy} = 0$

$$\left(f_{xx} f_{yy} - f_{xy}^2\right)\Big|_{\substack{x=-1 \\ y=\frac{1}{2}}} = 16 > 0.$$

$$f_{xx}(-1, \tfrac{1}{2}) = -2 < 0$$

$\boxed{(-1, \tfrac{1}{2}) \text{ is a local maximum point}}$.