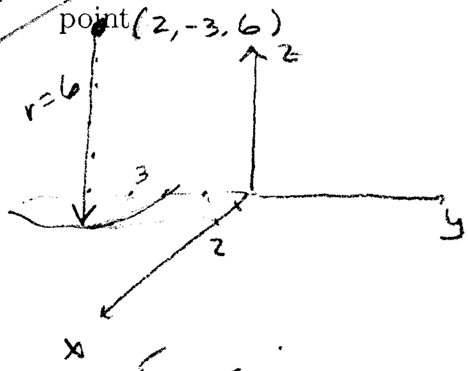
Name: *Laura Gross*

13.1 HW21(a)

1. Find the equation of a sphere with center $(2, -3, 6)$ that touches the xy-plane in exactly one point **10 pts**

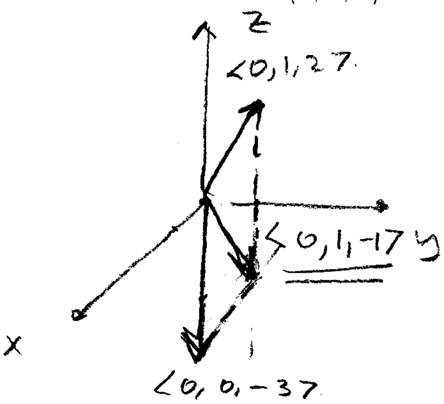


point is 6 units from xy-plane $\rightarrow$ $r = 6$

$$(x-2)^2 + (y+3)^2 + (z-6)^2 = 36$$

13.2 HW15

2. State the sum of the vectors $\langle 0, 1, 2\rangle$ and $\langle 0, 0, -3\rangle$ and illustrate the vector addition geometrically. **10 pts**



3. Consider the vector $\mathbf{v} = \langle 0, 1, -1\rangle$.

like part of 13.4 1-7,13

(a) If $\mathbf{w} = \langle 3, 0, 1\rangle$, find $\mathbf{v} \times \mathbf{w}$. **6 pts**

$$\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 1 & -1 \\ 3 & 0 & 1 \end{vmatrix} = \langle 1-0, -(0+3), 0-3\rangle$$

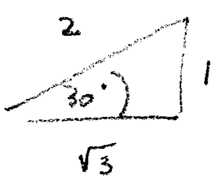
$$= \langle 1, -3, -3\rangle$$

like part of 13.4 HW 10

(b) If $\mathbf{u}$ is a vector of length 4 that makes a 30-degree angle with $\mathbf{v}$, what is the **exact** length of $\mathbf{v} \times \mathbf{u}$? **6 pts**

$$\|\underline{v}\|\|\underline{u}\| \sin 30° = \sqrt{0^2+1^2+(-1)^2}\; 4 \; \frac{1}{2}$$

$$= \sqrt{2} \cdot 2$$



(OVER)

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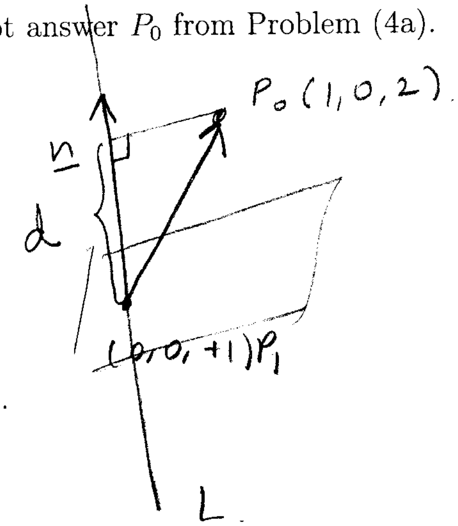
4. Consider the plane given by $3x + 2y - z + 1 = 0$. As you work the problem below, draw a thorough sketch.
   3 $-2+1 \neq 0$

(*) (a) Does $P_0(1, 0, 2)$ lie on the plane? Circle one: YES **NO** — 2 pts

(*) (b) Find a point P_1 on the plane. Do not answer P_0 from Problem (4a). — 2 pts

$(0, 0, +1)\ P_1$



~~13.2~~ (c) Find the vector $\vec{P_1P_0}$. — 2 pts

$\langle 1, 0, 1 \rangle$

(*) (d) Find a normal vector $\mathbf{n}$ to the plane. — 2 pts

$\underline{n} = \langle 3, 2, -1 \rangle$

~~like 13.3 35-40~~ (e) Find the scalar projection of $\vec{P_1P_0}$ onto $\mathbf{n}$. Take the absolute value. — 6 pts

$$\frac{\langle 1, 0, 1 \rangle \cdot \langle 3, 2, -1 \rangle}{\| \langle 3, 2, -1 \rangle \|} = \frac{3 - 1}{\sqrt{9+4+1}} = \frac{2}{\sqrt{14}}$$

$$= d$$

~~discussed extensively in class notes~~ (f) Interpret the number you found in (4e) geometrically in one complete sentence. — 4 pts

It is the distance from P_0 to the plane.

~~like 13.5 HW5~~ (g) State parametric equations for the line through P_0 that is perpendicular to the plane. — 6 pts

L:
$x = 1 + 3t$
$y = 0 + 2t$
$z = 2 + (-1)t$

(*) basic 13.5

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like # part of 13.6 11-20

Find the traces of the surface $z = \sqrt{36 - 9x^2 - 4y^2}$ in the planes $z = k$. Identify the surface, and sketch it.

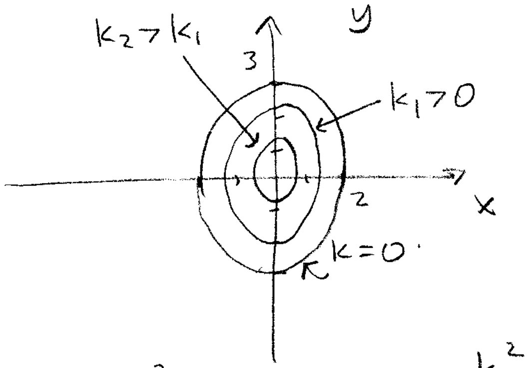
10 pts

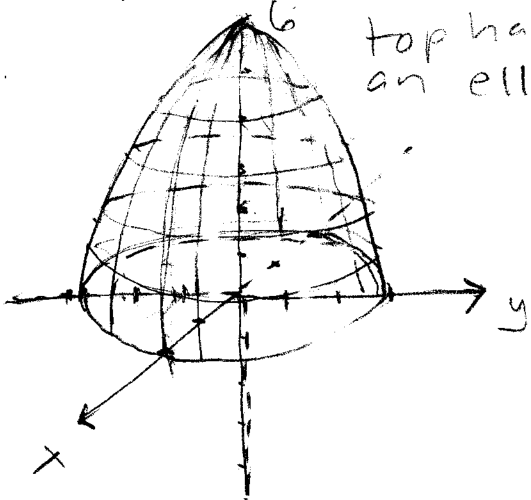
Note $z \geq 0$

$$z^2 = 36 - 9x^2 - 4y^2$$

$$\frac{9x^2 + 4y^2 + z^2 = 36}{36}$$

$$\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{36} = 1.$$





top half of an ellipsoid

$$\frac{x^2}{4} + \frac{y^2}{9} = 1 - \frac{k^2}{36}.$$

13.7 HW 57

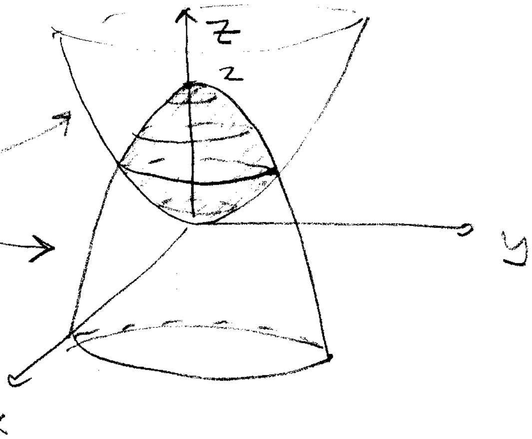
5. Sketch and/or fully describe the solid defined by the given inequalities.

(a) $r^2 \leq z \leq 2 - r^2$

6 pts

$z = r^2$ $\quad$ $z = 2 - r^2$

$z = x^2 + y^2$ $\quad$ $z = 2 - x^2 - y^2$



13.7, 59

(b) $\rho \leq 2,\ 0 \leq \phi \leq \frac{\pi}{2},\ 0 \leq \theta \leq \frac{\pi}{2}$: portion of the [solid sphere of radius 2 centered at the origin] that lies in the first octant.

6 pts

(OVER)

14.1, 5 ~~#~~ ~~Find~~ $\lim_{t\to 1}\left(\sqrt{t+3}\,\mathbf{i}+\frac{t-1}{t^2-1}\,\mathbf{j}+\frac{\tan t}{t}\,\mathbf{k}\right)$. $= \lim_{t\to 1}\left(\sqrt{t+3}\;\underline{i} + \frac{\cancel{t-1}}{(\cancel{t-1})(t+1)}\,\underline{j} + \frac{\tan t}{t}\,\underline{k}\right.$

10 pts

$= \left\langle \sqrt{4},\ \frac{1}{2},\ \frac{\tan(1)}{1} \right\rangle$

$= \left\langle 2,\ \frac{1}{2},\ \tan(1) \right\rangle$

6. Consider the vector function $\mathbf{r}'(t) = t^2\mathbf{i} + 4t^3\mathbf{j} - t^2\mathbf{k}$. (Note this is $\mathbf{r}$ *prime*.)

like 14.2 9–16

(a) Find $\mathbf{r}''(t)$. $= \langle 2t,\ 12t^2,\ -2t \rangle$

6 pts

14.2 HW 39 (b) ~~Find~~ $\mathbf{r}(t)$ if $\mathbf{r}(0) = \mathbf{j}$.

6 pts

$\underline{r} = \left\langle \frac{t^3}{3},\ t^4,\ -\frac{t^3}{3} \right\rangle + \underline{c}$

$\underline{r}(0) = \underline{0} + \underline{c} = \underline{j} \rightarrow \underline{c} = \underline{j}$

$\underline{r} = \left\langle \frac{t^3}{3},\ t^4,\ -\frac{t^3}{3} \right\rangle + \langle 0, 1, 0 \rangle$

$= \left\langle \frac{t^3}{3},\ t^4+1,\ -\frac{t^3}{3} \right\rangle$