Name: ________________________________

1. Determine **and also sketch** the set of points at which the function $F(x, y) = \arctan(x + \sqrt{y})$ is continuous.

9 pts

2. Use implicit differentiation to find $\partial z/\partial y$ if $yz = \ln(x + z)$. (Here z is a function of x and y.)

9 pts

3. Evaluate the double integral $\int\int_R (4 - 2y)\, dA$, $R = [-1, 3] \times [0, 1]$ **by first identifying it as the volume of a solid**. (Note: You will not receive partial credit for evaluating the integral using iterated integrals as in §16.2. That is, **do not** use antiderivatives. This question tests your understanding of a double integral as a volume. As such, **illustrate your answer with a quick graph**.)

9 pts

4. Use a tree diagram to write out the chain rule for $\partial M/\partial v$ if $M = f(x, y, z)$, $x = x(u, v)$, $y = y(u, v)$, $z = z(u, v)$. (Assume all functions are differentiable.)

9 pts

5. Let $M = xe^{y-z^2}$, $x = 2uv$, $y = u - v$, $z = u + 4v$. Find an **exact value** for $\partial M/\partial v$ at $u = 3$, $v = -1$. (You can use your answer to the previous problem as a starting point.)

9 pts

6. Find the linearization of the function $f(x, y) = \sin(2x + 3y)$ at the point $(\pi/8, 0)$.

10 pts

7. Find the directional derivative of $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$ at the point $(1, 2, -2)$ in the direction of the vector $\mathbf{v} = < -6, 6, -3 >$.

9 pts

8. Consider $f(x, y) = x^2 + 4y^2$.

   (a) Find the normal direction to the level curve $f(x, y) = 8$ at the point $(2, 1)$. [4 pts]

   (b) Sketch the level curve through the point $(2, 1)$ and the gradient vector at the point $(2, 1)$ on the same set of axes, showing the scaling on the axes. [6 pts]

   (c) Find the maximum rate of change of f at the point $(2, 1)$. [4 pts]

   (d) In what direction does the maximum rate of change of f occur at the point $(2, 1)$? (That is, find the direction of steepest ascent of the function.) [4 pts]

9. Find all the **critical points** of the function $x^4 + y^4 - 4xy + 2$. (**DO NOT** classify the critical points as maxima, minima, or saddles; just find the critical points.)

9 pts

10. Consider the function $f(x, y) = x^3 - 6xy + 8y^3$.

    (a) Verify that $(1, 1/2)$ is a critical point.

2 pts

    (b) Classify $(1, 1/2)$ as a local maximum, local minumum, or saddle point using the Second Derivatives Test.

7 pts