1. Consider the vector function $\mathbf{r} = 2\sin t\mathbf{i} + 3\cos t\mathbf{j}$.

   (a) Sketch the plane curve. (Include an arrow to show the direction of increasing t.

10 pts

   (b) Find the exact unit tangent vector at $t = \pi/3$.

10 pts

   (c) Find the exact curvature at $t = \pi/3$.

10 pts

(OVER)

2. A particle in space undergoes constant nonzero acceleration. The initial velocity is $\mathbf{0}$.

   (a) Find the particle's trajectory.

   5 pts

   (b) The trajectory is (circle one):

   a line   a parabola   neither a line nor a parabola.

   5 pts

3. Consider $\displaystyle\lim_{(x,y)\to(0,0)} \frac{2xy}{x^2+2y^2}$.

   (a) Find the value(s) that $\dfrac{2xy}{x^2+2y^2}$ approaches when $(x, y) \to (0, 0)$ along straight-line paths.

   5 pts

   (b) The limit (circle one):

   5 pts

   i. exists,
   ii. does not exist,
   iii. cannot be determined to exist or not to exist from the information in (3a).

4. Use the chain rule to find $\dfrac{\partial z}{\partial u}$ if $z = x^2 + xy^3$, $x = uv^2 + w^3$, and y $= u + ve^w$.

   10 pts

5. Consider $f(x, y) = \sqrt{x^2 + y^2}$.

   (a) Calculate $\nabla f(1, 1)$.

7 pts

   (b) Sketch $\nabla f(1, 1)$, showing its orthogonality to the relevant level curve. Label the axes, and show the scale.

6 pts

   (c) Find the rate of change of f in the direction $\mathbf{i} + 2\mathbf{j}$ at $(1, 1)$.

7 pts

(OVER)

6. Find the equation of the tangent plane to the surface $25x^2 + 9y^2 + 16z^2 = 41$ through the point $(1, 0, 1)$.

10 pts

7. Find the absolute maximum and miniumum values of the function $f(x, y) = e^{-x^2-y^2}(x^2 + 2y^2)$ on the set $D = \{(x, y)|x^2 + y^2 \leq 4\}$, given that the only critical points of f inside D are $(0, 0)$, $(\pm 1, 0)$, and $(0, \pm 1)$ and given that $f(0, 0) = 0$, $f(\pm 1, 0) = 1/e$, and $f(0, \pm 1) = 2/e$.

10 pts