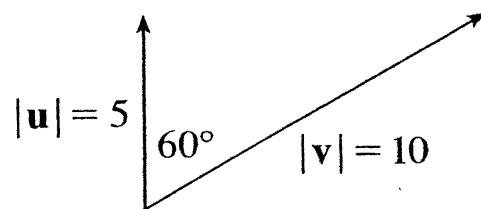


1. The figure shows the vectors \mathbf{u} and \mathbf{v} .

12 pts



(a) Draw $\mathbf{v} - \mathbf{u}$ on the figure.

(b) Find the exact value of $|\mathbf{u} \times \mathbf{v}|$:

(c) Determine whether $\mathbf{u} \times \mathbf{v}$ is directed INTO the page or OUT OF the page. Circle one:
INTO **OUT OF**

2. Using the definition of the cross product, prove $\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$ for the vectors $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$. (Note you are proving this for all possible vectors in 3-D, so do **not** give example values for the components a_i and b_i .)

8 pts

(OVER)

3. Find the equation of the normal plane of the curve $x = 2 \sin(3t)$, $y = t$, $z = 2 \cos(3t)$ at the point $(0, \pi, -2)$.

8 pts

4. Find $\partial R / \partial x$ when $x = y = 1$ if $R = \ln(u^2 + v^2 + w^2)$, $u = x + 2y$, $v = 2x - y$, and $w = 2xy$.

8 pts

5. Find the equation of the normal line to the surface given by $z + 1 = xe^y \cos z$ at the point $(1, 0, 0)$.

8 pts

6. Find the (exact) *surface area* of the part of the plane $3x + 2y + z = 6$ that lies in the first octant.

15 pts

(OVER)

7. Consider the particle whose path is given by

20 pts

$$\mathbf{r}(t) = \cos(t) \mathbf{i} + \sin(t) \mathbf{j} + t \mathbf{k}.$$

(a) Sketch the space curve with the given vector equation. Indicate with an arrow the direction in which t increases.

(b) Find the acceleration vector for the particle.

(c) Find a tangent vector to the particle path, and show that this tangent vector is orthogonal to the acceleration vector you found in (7b).

(d) Find the normal component of the acceleration vector you found in (7b).

8. Find the point on the plane $x - y + z = 4$ that is closest to the point $(1, 2, 3)$ (using max/min techniques from §15.7). Show your work.

15 pts

(OVER)

9. Set up **but do not evaluate** an integral for the volume of the solid above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = 1$.

8 pts

10. Consider the solid E whose volume is given by $\int_0^{\pi/6} \int_0^{\pi/2} \int_0^3 \rho^2 \sin \phi \, d\rho d\theta d\phi$.

- (a) Sketch the solid E , labeling key values of ρ , θ , and ϕ . Briefly describe the region in words.

8 pts

- (b) Evaluate the integral.

7 pts

11. Evaluate $\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$ if $\mathbf{F} = x^3y^4 \mathbf{i} + x^4y^3 \mathbf{j}$, and \mathcal{C} is given by $\mathbf{r} = \sqrt{t} \mathbf{i} + (1 + t^3) \mathbf{j}$, $0 \leq t \leq 1$.

8 pts

12. Evaluate $\int_{\mathcal{C}} x dx + y dy$ if \mathcal{C} consists of the line segments from $(0, 1)$ to $(0, 0)$ and from $(0, 0)$ to $(1, 0)$ and the parabola $y = 1 - x^2$ from $(1, 0)$ to $(0, 1)$. Include a sketch of \mathcal{C} .

8 pts

(OVER)

13. **Extra Credit:** Calculate (a) the curl and (b) the divergence of the vector field

$$\mathbf{F}(x, y, z) = \langle xe^{-y}, xz, ze^y \rangle.$$

8 pts