3450:439/539:001 **Homework 7** Spring 2008

Due: Thursday, March 6, 2008

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1. Consider the eigenvalue problem $x^2y''(x)+xy'(x)+x^2\lambda y(x) = 0$, $0 < x < 1$, $y(1) = 0$ and $\lim_{x\to 0^+} |y(x)| < \infty$. In this problem exploit the work you already did on Homeworks 5 and 4.

   (a) Assuming that the boundary condition at zero is "appropriate," show that the problem is a singular Sturm-Liouville boundary-value problem.

   (b) Find the eigenvalues and eigenfunctions.

   (c) Show that eigenvalues for the problem have the properties guaranteed by the Sturm-Liouville Theorem. You can illustrate many of the properties with a quick sketch of a Bessel function as sketched in class.

   (d) The eigenfunctions $y_n(x)$, $y_m(x)$ $(n \neq m)$ satisfy the orthogonality relation $\langle y_n(x), y_m(x)\rangle = 0$. Define the inner product $\langle y_n(x), y_m(x)\rangle$ in terms of the appropriate integral.

   (e) Prove the eigenfunctions satisfy $\langle y_n(x), y_m(x)\rangle = 0$ for the inner product you defined.

2. Find the Fourier series of

$$f(x) = \begin{cases} 2, & -2 \leq x < 0 \\ x, & 0 \leq x < 2. \end{cases} .$$