

## 3450:439/539:001 **Homework 3** Spring 2008

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Due: Thursday, January 31, 2008

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1. For each equation ( $x > 0$ ) solve directly without the explicit use of series, OR find the first five nonzero terms of the power-series expansion about zero of the general solution (showing  $x = 0$  is an ordinary point), OR (i) show that zero is a regular singular point, (ii) find and solve the indicial equation, (iii) determine the recurrence relation, and (iv) use the results of (ii) and (iii) to find the first five nonzero terms of two linearly independent Frobenius solutions.

(a)  $xy'' + (1 - x)y' + y = 0$

(b)  $x(x - 1)y'' + 3y' - 2y = 0$

(c)  $x^2y'' + 7xy' + 13y = 0$

(d)  $x^2y'' + xy' + x^2y = 0$

2. Consider the equation

$$x^2y'' + \left( \alpha^2\beta^2x^{2\beta} + \frac{1}{4} - \nu^2\beta^2 \right) y = 0, \quad x > 0.$$

- (a) Verify it has a solution of the form  $y = x^{1/2}f(\alpha x^\beta)$ , where  $f$  is a solution of Bessel's equation of order  $\nu$ .
- (b) Write the general solution to

$$x^2y'' + (4x^2 - 2)y = 0, \quad x > 0.$$