

Due: Thursday, January 24, 2008

1. Solve the boundary-value problem (BVP)

$$\Psi''(x) = 0 \quad (0 < x < L), \quad \Psi(0) = A, \quad \Psi'(L) = 0,$$

where A and L are prescribed numbers.

2. Solve the boundary-value problem (BVP)

$$\kappa\Psi''(x) + e^x = 0 \quad (0 < x < L), \quad \Psi(0) = 0, \quad \Psi(L) = 0,$$

where κ and L are prescribed numbers.

3. Consider the constant-coefficient ODE $au''(x) + bu'(x) + cu = 0$ with $b^2 - 4ac = 0$.

- (a) Find a real-valued solution of the form $u_1(x) = e^{rx}$.
(b) The method of reduction of order says that if $u_1(x)$ is a solution to the variable-coefficient ODE $u''(x) + p(x)u'(x) + q(x)u(x) = 0$, then a second solution has the form

$$u_2(x) = u_1(x)v(x), \quad \text{where} \quad v(x) = \int \frac{\exp\left[-\int p(x) dx\right]}{[u_1(x)]^2} dx.$$

Find $u_2(x)$ using the method of reduction of order. (Show $u_2(x)$ has the same form that we used in class.)

4. Calculate $\lim_{\omega \rightarrow \infty} \rho e^{-i\omega}$ for $\omega \in \mathbb{R}$, or state that the limit does not exist. Show your work using Euler's formula $e^{i\theta} = \cos \theta + i \sin \theta$.
5. Suppose that $f(x)$ can be expanded into a series of the form

$$\sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi}{L}x\right).$$

- (a) Write out the forms of the first five terms.
(b) If $f(x) = \sin\left(\frac{\pi}{L}x\right) - 3\sin\left(\frac{4\pi}{L}x\right)$, what are the values of b_n , $n = 1, 2, \dots$?
(Note: Once you understand the question, the answer is so easy as to seem ridiculous.)
6. Find the first five nonzero terms of the power-series expansion about zero of the general solution to

$$y'' - 2y' + xy = 0.$$