# 3450:439/539:001 **Homework 13** Spring 2008

Recommended due date: Thursday, April 24, 2008

**THIS HOMEWORK IS NOT FOR COURSE CREDIT.** However, you need to do problems to learn the material. Also, about 1/3 of your exam will consist of recommended homework problems.

1. Solve the initial-boundary-value problem below, where the forcing term $F(x,t)$ has $t$-dependence.

$$\begin{aligned} \frac{\partial^2 y}{\partial t^2} &= c^2 \frac{\partial^2 y}{\partial x^2} + F(x,t), \quad 0 < x < L, \quad t > 0, \\ y(0,t) = 0, \quad y(L,t) &= 0, \qquad\qquad t > 0, \\ y(x,0) = f(x), \quad \frac{\partial y}{\partial t}(x,0) &= 0, \qquad\qquad 0 < x < L. \end{aligned}$$

2. Consider the problem

$$\begin{aligned} \frac{\partial u}{\partial t} &= \kappa \frac{\partial^2 u}{\partial x^2}, \qquad -\infty < x < \infty, \quad t > 0 \\ u(x,0) &= f(x), \qquad -\infty < x < \infty, \end{aligned}$$

where

$$f(x) = \begin{cases} e^{-x}, & -1 \le x \le 1 \\ 0, & |x| > 1 \end{cases}.$$

If the diffusivity constant $\kappa = 1$, graph the temperature distribution over an appropriate $x$ interval for representative values of $t$ using a graphing calculator, Mathcad, etc. Give a brief physical interpretation of the graphs.

3. Consider the problem

$$\begin{aligned} \frac{\partial u}{\partial t} &= \kappa \frac{\partial^2 u}{\partial x^2}, \qquad x > 0, \quad t > 0 \\ u(0,t) &= t^2, \quad t \ge 0 \\ u(x,0) &= 0, \quad x > 0. \end{aligned}$$

(a) Obtain a solution using the Laplace transform.

(b) If the diffusivity constant $\kappa = 1$, graph the temperature distribution over an appropriate $x$ interval for representative values of $t$ using a graphing calculator, Mathcad, etc. Give brief a brief physical interpretation of the graphs.

**(OVER)**

4. Use Laplace transform to solve

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}, \qquad x > 0, \quad t > 0,$$

$$y(0,t) = \begin{cases} \sin(2\pi t), & 0 \le t \le 1, \\ 0, & t > 1, \end{cases}$$

$$y(x,0) = 0, \quad \frac{\partial y}{\partial t}(x,0) = 0, \qquad x > 0.$$

   This problem models the displacement of a string initially at rest along the positive $x$-axis, with the left end moving in the prescribed fashion. Imagine, for example, a long jump rope, with a child at the left end moving the rope periodically. Determine the solution that decays to zero with increasing $x$ for all time.

5. Solve the following initial-boundary-value problem using the method of Fourier transform.

$$\frac{\partial^2 u}{\partial t^2} = 144 \frac{\partial^2 u}{\partial x^2}, \qquad -\infty < x < \infty, \quad t > 0$$

$$u(x,0) = e^{-5|x|}, \qquad -\infty < x < \infty,$$

$$\frac{\partial u}{\partial t}(x,0) = 0, \qquad -\infty < x < \infty.$$

6. Solve the following boundary-value problem using the method of Fourier transform.

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \qquad -\infty < x < \infty, \quad 0 < y < 1,$$

$$u(x,0) = \begin{cases} 0, & x < 0, \\ e^{-ax}, & x > 0, \end{cases}$$

$$u(x,1) = 0, \qquad -\infty < x < \infty.$$