3450:439/539:001 **Homework 12** Spring 2008

Due: Thursday, April 17, 2008

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1. Solve

$$\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2} + 2xt, \qquad -\infty < x < \infty, \quad t > 0,$$
$$u(x,0) = \sin x, \quad \frac{\partial u}{\partial t}(x,0) = 2x, \quad -\infty < x < \infty.$$

2. In class we solved the initial-boundary-value problem

$$\frac{\partial u}{\partial t} = \kappa\frac{\partial^2 u}{\partial x^2}, \quad 0 < x < L, \quad t > 0,$$
$$u(0,t) = A, \quad \frac{\partial u}{\partial x}(L,t) = 0, \qquad t > 0,$$
$$u(x,0) = f(x), \qquad 0 < x < L.$$

Consider the case in which $\kappa = 4$, $A = 2$, $L = 6$, $f(x) = x(6-x) + 2$.

(a) Graph the function $f(x) = x(6-x) + 2$ on the relevant interval. Give a physical interpretation of the function.

(b) Find the fourth partial sum of the series representation of $u(x,t)$, and graph it for representative values of t using a graphing calculator, Mathcad, etc. Give brief physical interpretations for the graphs.

(c) Describe how you could judge whether four terms is an appropriate number to include in a good partial-sum approximation of the series solution. Optional: Carry out your method and draw conclusions.

(d) Using your approximation in (2b), what is the temperature of the bar at its midpoint after three units of time?

(e) What is the temperature distribution in the bar as $t \to \infty$? Answer the question from both the mathematical and physical standpoints.

3. Consider a uniform thin string, whose length at rest is 10 cm. Assume that the end at $x = 0$ is anchored at a vertical displacement of zero, while the end $x = 10$ is held at a vertical displacement of 1 cm. If the string's initial configuration is given by $f(x)$ and the intial velocity is zero, find a series representation for the configuration of the string for all time. Express the coefficients in integral form.

(OVER)

4. Solve the initial-boundary-value problem

$$\begin{aligned} \frac{\partial^2 y}{\partial t^2} &= 3\frac{\partial^2 y}{\partial x^2} + 2x, \quad 0 < x < 2, \quad t > 0, \\ y(0,t) = 0, \quad y(2,t) &= 0, \qquad\qquad t > 0, \\ y(x,0) = 0, \quad \frac{\partial y}{\partial t}(x,0) &= 0, \qquad\qquad 0 < x < 2. \end{aligned}$$

5. Use the idea of Homework 1, Problem 9 or separation of variables to solve

$$\begin{aligned} \frac{\partial u}{\partial t} &= \frac{\partial^2 u}{\partial x^2} + 6\frac{\partial u}{\partial x}, \quad 0 < x < 4, \quad t > 0, \\ u(0,t) = 0, \quad u(4,t) &= 0, \qquad\qquad t > 0, \\ u(x,0) &= 1, \qquad\qquad 0 < x < 4. \end{aligned}$$