HW2 - A Sol'n

AEM II
Gross
S2008.

(1) $\psi' = c_1$

$\psi = c_1 x + c_2$.

$\psi(0) = c_2 = A$

$\psi'(L) = c_1 = 0$.

$$\boxed{\psi(x) \equiv A}$$

(2). $\psi'' = -\frac{e^x}{K}$.

$\psi' = -\frac{1}{K} e^x + c_1$

$\psi = -\frac{1}{K} e^x + c_1 x + c_2$.

$\psi(0) = -\frac{1}{K} + c_2 = 0$

$\rightarrow c_2 = \frac{1}{K}$.

$\psi = -\frac{1}{K} e^x + c_1 x + \frac{1}{K}$.

$\psi(L) = -\frac{1}{K} e^L + c_1 L + \frac{1}{K} = 0$.

$\rightarrow c_1 = \frac{\frac{1}{K}(e^L - 1)}{L}$.

$$\boxed{\psi = -\frac{1}{K} e^x + \frac{1}{KL}(e^L - 1) x + \frac{1}{K}}$$

(3). (a) $ar^2 + br + c = 0 \rightarrow r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-b}{2a}$

$u_1(x) = e^{-\frac{b}{2a} x}$.

(b). $v(x) = \int \frac{e^{-\int \frac{b}{a} dx}}{(e^{-\frac{b}{2a} x})^2} dx = \int \frac{e^{-\frac{b}{a} x}}{e^{-\frac{b}{a} x}} dx$

$= \int dx = x$.

$u_2(x) = e^{-\frac{b}{2a} x} x$.

(4). $\lim_{\omega\to\infty} \rho e^{-i\omega} = \lim_{\omega\to\infty} \rho(\cos\omega - i\sin\omega)$ HW2, P2.

does not exist!

(5). (a) $b_1 \sin\frac{\pi x}{L} + b_2 \sin\frac{2\pi x}{L} + b_3 \sin\frac{3\pi x}{L} + b_4 \sin\frac{4\pi x}{L}$

$+ b_5 \sin\frac{5\pi x}{L} + \cdots$

(b) $= \sin\frac{\pi x}{L} - 3\sin\frac{4\pi x}{L}$ if

$b_1 = 1,\ b_2 = 0,\ b_3 = 0,\ b_4 = -3,\ b_n = 0,\ n \geq 5$

(6). $y'' - 2y' + xy = 0.$ $y = \sum_{n=0}^{\infty} a_n x^n$ ($\frac{Q}{P} = -2$, $\frac{R}{P} = x$ are their own pow-ser. expans)

$$\sum_{n=2}^{\infty} n(n-1)a_n x^{n-2} + \sum_{n=1}^{\infty} -2n a_n x^{n-1} + \sum_{n=0}^{\infty} a_n x^{n+1} = 0.$$

$m = n-2$. $m = n-1$ $m = n+1$

$$\sum_{m=0}^{\infty} (m+2)(m+1) a_{m+2} x^m + \sum_{m=0}^{\infty} -2(m+1) a_{m+1} x^m +$$

$$+ \sum_{m=1}^{\infty} a_{m-1} x^m = 0.$$

$m=0$: $2a_2 - 2a_1 = 0. \to \boxed{a_2 = a_1}$

$m \geq 1$: $(m+2)(m+1)a_{m+2} - 2(m+1)a_{m+1} + a_{m-1} = 0.$

$$a_{m+2} = \frac{2\cancel{(m+1)}\, a_{m+1}}{(m+2)\cancel{(m+1)}} - \frac{a_{m-1}}{(m+2)(m+1)}.$$

$m=1$: $a_3 = \frac{2}{3} a_2 - \frac{1}{6} a_0 = \boxed{\frac{2}{3} a_1 - \frac{1}{6} a_0 = a_3}$

$m=2$: $a_4 = \frac{2}{4} a_3 - \frac{1}{12} a_1 = \frac{1}{2}\left(\frac{2}{3}a_1 - \frac{1}{6}a_0\right) - \frac{1}{12}a_1$

$= \boxed{\frac{1}{4} a_1 - \frac{1}{12} a_0 = a_4}$

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + \cdots$$

$$= a_0 + a_1 x + a_1 x^2 + \left(\frac{2}{3} a_1 - \frac{1}{6} a_0\right) x^3 + $$

$$+ \left(\frac{1}{4} a_1 - \frac{1}{12} a_0\right) x^4 + \cdots$$

$$= a_0 \left[1 - \frac{1}{6} x^3 - \frac{1}{12} x^4 + \cdots\right] +$$

$$+ a_1 \left[x + x^2 + \frac{2}{3} x^3 + \frac{1}{4} x^4 + \cdots\right].$$