

THE UNIVERSITY OF AKRON  
Mathematics and Computer Science



e-calculus  
homepage

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# Introduction

Patience  
Dedication  
Concentration  
Supreme Concern

It is, perhaps, the height of arrogance to think that one person, or any group of people, can write a set of tutorials that can *cause* the reader to, at last, come out of the darkness and into the light of knowledge of mathematics. Is it possible to explain the concepts and illustrate them through examples in just the right way that will enable the reader to, at last, understand the mathematical ideas that have alluded him all these years? By the eloquence of these tutorials, can the love of mathematics and learning be conveyed to the student?

During the last decade of the 20<sup>th</sup> century, there has been a phenomenally rapid change in computer technology, both in software and hardware. Processors have become more powerful, hard-drives have vastly increased capacity, RAM is faster and less expensive; taking advantage of hardware improvements, software has become sophisticated, intelligent and user-friendly. Throughout this period, professional educators have been continually striving to understand how to

incorporate these new technologies into an academic setting. Mathematics poses a number of special problems in this regard. These tutorials then are one such attempt at applying computer technologies to mathematical education.

To begin with, mathematics is such a vast field that no tutorial or set of tutorials created by one person can ever hope to come close to being comprehensive. Nevertheless, I will try to provide a modicum of information on diverse topics; I will try to provide insights into how to think about mathematical concepts, how ideas are organized, and how general principles are applied in problem solving. I shall try to suggest ideas for you to think about, I shall ask you to create your own ideas—to explore mathematics on your own. If I have peaked your interest, challenged your mind, expanded your knowledge, and created within you a love of learning, study, and rational thinking, then I will have done a good days work, and in the process, learned a lot myself.

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## Game Section

### Directory

- **The Giants of Calculus**

A two-column matching game. The game tests your knowledge of some of the great names in Calculus.

- **Algeboard!**

A “Jeopardy-like” game covering topics in (high school) algebra.

- **Algeboard.** This is the original version. No special setup is needed and can be viewed off-line.
- **Algeboard/JS.** This is the new JAVASCRIPT version and requires **Adobe Forms 3.5**. (See the [e-@calculus home-page](#) for more information on Forms 3.5.) In this version, the game keeps your score as you play.

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## Essays

- How to Use this Tutorial.
- The Tutorial's Role in the Classroom.
- The Key to Success? It starts with an 'A'.
- Math Tests: Nothing to Fear!

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# How to Use this Tutorial

**My advice: Read, read, read, read, and, of course, read!**

e-@lculus is designed for the serious student of mathematics. (This should include engineering majors.) Students who are willing to devote large amounts of time to the study of Calculus. Calculus cannot be learned through casual study, through quick summaries of topics, by flashy graphics and animations, or by well-scripted sound bites, it can only be mastered by a slow, methodical, deliberate, thoughtful study of the ideas. e-@lculus, hopefully, lays out a series of lessons for the student to pursue the latter course of study.

e-@lculus can be used as a supplementary resource to a traditional course in Calculus. [The Tutorial's Role in the Classroom](#) describes some ideas for incorporating these tutorials/document preview system into the classroom.

**Why e-Calculus and not p-Calculus?**<sup>1</sup> e-Calculus attempts to give a *verbose* discussion of the major topics of *Calculus I*. Such a discussion is normally not available in a traditional paper-bound text.

Beyond the verbosity of the discussions, the advantages of using the tutorial are several. It is a source of additional examples. The exercises, with solutions, can aid you in your problem solving. There are in-line quizzes to test your understanding. There are graphics, both static and dynamic, to clarify concepts that a mere textbook cannot do.

**Concerning the Exercises.** There are four obvious options open to you when you encounter an **Exercise**:

1. Skip over it. (Bad choice.)
2. Treat it as an **Example**. (Less bad than the previous bad.)
3. Try to solve the problem where you sit. (Good choice.)

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<sup>1</sup>p-Calculus = paper Calculus or pulp Calculus

4. Copy the problem down for solution later. Solve it and return to my tutorial for verification of the total correctness of your solution. (Maybe the best choice?)
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# The Tutorial's Role in the Classroom

To put it simply—I don't know. This tutorial, I believe, is the only one of its kind. It may be true that this kind of delivery system will be the textbook of the future. There will come a time when hand-held electronic “readers” will be used; at that time, a tutorial such as mine (which, by the way, is practically an on-line calculus book without the “ $\infty - \epsilon$ ” number of exercises) can be carried around like a book. An electronic format offers the advantage of some degree of interactivity, as you will see, as well as the ability of deliver static and animated graphics.

Until that time, e-@lculus can and will act as a passive tutorial in my classes. (Just as the textbook is.) Given that you don't want to coerce the student into using the tutorial, the question then becomes how to encourage the student to be involved with an on-line system.

Here are some ideas that I put forth as ways in which the student is encouraged to get involved with the tutorial. Except for putting solutions to tests on-line, I have tried and am still using these methods.

- *During Lectures.* Make frequent references to the tutorial during lecture—suggesting to the class that more information, examples, and exercises can be obtained on-line.

- *Lecture Notes.* Put the gist of the day's lectures on-line—in summary form. Cross-reference the days topics with the corresponding topics in the tutorials.

- *Tests.* Solutions to tests can be placed on-line.

- *Quizzes.* Some teachers give regular quizzes. The quizzes along with solutions can be placed on-line for the student to peruse.

- *Weekend Problems.* One thing that I did was to hand out a set of weekend problems each Friday (in addition to regular homework assignments). The number of problems was not great and covered the material discussed in the past week.

Ideally, the students solved the problems over the weekend. The following Monday, the solutions to the problems were put *on-line* for the students to read and to compare with their own solutions.

Before reading the solutions, however, I presented an on-line *quiz* for them to take. If they had solved the problems, they should get 100% on each of the quizzes. This would give the student immediate feedback on their success. After the quiz (for no credit) they could then go on to read the solutions of the ones missed—or the right ones for that matter.

- *Hooks to Computer Algebra Systems.* On-line tutorials in the use of the CAS popular at your site can be created. Hypertext links to prepared “worksheets” can be included in the tutorials.

By the way, I have also written a fairly detailed tutorial over [Maple V, Version 3](#). It was a tutorial that paralleled e-[Calculus: Functions, Limits, Differentiation, and Integration](#). With the introduction of [Maple V, Version 4](#)—a major revision—the [Maple](#) tutorial went immediately out of date. This dampened my enthusiasm for writing tutorials for software constantly undergoing revision. ■

Some of the above can be done using `html` documents. For those who use  $\text{T}_{\text{E}}\text{X}$  or  $\text{L}_{\text{A}}\text{T}_{\text{E}}\text{X}$  to produce course materials, the purchase of the

**Adobe Acrobat** would enable you to put typeset quality mathematics on-line.

Should you have an interest in developing hypertext link capabilities within your  $\text{T}_{\text{E}}\text{X}$ /PDF documents, they can either be created by “hand” using the **Acrobat Exchange**, or they can be automatically created using `pdfmarks` within the Postscript output file. The latter is done using  $\text{T}_{\text{E}}\text{X}$  macros and is the method that I use.

At a later time, I shall make available the macros that I developed to create these tutorials—should there ever be any interest by anyone out there in cyberland. These macros, though, were designed to work with the special `\special's` of the [Y&Y  \$\text{T}\_{\text{E}}\text{X}\$  System](#). Alternatively, there are freeware  $\text{T}_{\text{E}}\text{X}$  systems out there that have color and hypertext capabilities.

One drawback to the freeware systems is the lack of a DVI viewer that shows color and responds to hypertext links. (There may be a UNIX DVI viewer available.) In the PC/Mac world of computing, there is no such DVI viewer, or, at least, none that I am aware of.

Why is this important? It is very useful to have a DVI previewer that has color and hypertext capabilities because the previewer can be used as a debugging tool for the PDF file. There is a long turn-around between  $\text{T}_{\text{E}}\text{X}$ ing and viewing a PDF file. All new links need to be checked; sometimes there are ‘color problems.’ The turn-around time from `tex` to `dvi` is considerably shorter than that of `tex` to `pdf`. Errors can be found and immediately fixed and the file can be previewed again. Only when all the ‘bugs’ are out do I finally `distill` the file.

Of course, the DVI viewer from [Y&Y](#) does follow hypertext links and display color; for this reason, the [Y&Y  \$\text{T}\_{\text{E}}\text{X}\$  System](#) turned out to be ideal for me.

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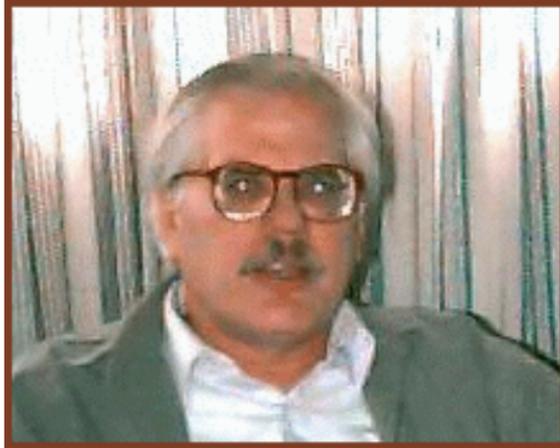


Photo by Mark Hamilton

**Thanks** are extended to the hard-disk of my monochrome screened IBM AT. By a lucky fluke of fate, by an act of sheer fortune, by a *Poisson event* of the very rarest kind . . . it crashed! Had it not crashed, I would not have gotten my current computer and this project would not even have been conceived.

DS

## Acknowledgements

The start up costs for this project were paid for by a grant from the EDUCATIONAL RESEARCH AND DEVELOPMENT OFFICE of **The University of Akron**.

I would like to acknowledge **Dr. David C. Buchthal**, the former<sup>2</sup> Chair of the Department of Mathematical Sciences, for his support of this project. He supplied me with very nice hardware that enabled me to T<sub>E</sub>X<sup>3</sup> and Distill<sup>4</sup> in a finite amount of time.

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<sup>2</sup>Dr. Buchthal's change in position is in no way related to this project; he has, in fact, been promoted to Associate Dean.

<sup>3</sup>T<sub>E</sub>X is a program created by **Donald Knuth** that enables me to produce typeset quality text and mathematics.

<sup>4</sup>Distill refers to the **Adobe Distiller**. This is the program that converts post-script files to portable document format files (**.pdf**).

# 1. Features of this Document Delivery System

For the casual browser, that's you, here is a quick tour of the some basic features of these tutorials. The disk based system has more capabilities than the web based system. Below are features common to each system.

## 1.1. In-line Examples and Exercises

You will find throughout the tutorials, numerous examples and exercises mixed in with the discussions.

Examples are stated but the solutions are displayed only if the student wants to see them.

**EXAMPLE 1.1.** Calculate  $\int_0^{\pi} \sin(x) dx$ .

Questions for the students are handled in the same way—through in-line exercises.

**EXERCISE 1.1.** Find the area under the graph of  $f(x) = \sin(x)$ ,  $0 \leq x \leq \pi$ .

Traditional to any calculus book is the listing of a large number of exercises at the end of each section. This eventuates into an ‘ $\infty - \epsilon$ ’ number of exercises. You’ll find none of that here. (At least not yet.)

## 1.2. Interactivity

Throughout the tutorials, there are multiple choice questions. There are several variations that are possible.

Below is an *immediate response question*.

**Quiz.** Let  $f(x) = 3x^2 + 2x + 1$ . Find the equation of the line tangent to the graph of  $f$  at  $x = 1$ .

- (a)  $y = 8x - 14$    (b)  $y = 8x - 2$    (c)  $y = 6x$    (d)  $y = 6x - 1$

**End Quiz.**

Another type of question is immediate response *with solution*.

**Quiz.** Let  $f(x) = 3x^2 + 2x + 1$ . Find the equation of the line tangent to the graph of  $f$  at  $x = 1$ .

- (a)  $y = 8x - 14$  (b)  $y = 8x - 2$  (c)  $y = 6x$  (d)  $y = 6x - 1$

**End Quiz.**

In the disk-based system, responses are all accompanied by annoying sounds.

In the disk-based system, it is possible to have graded quizzes. A background program keeps track of student responses and reports the score after the student has exited the quiz. After reporting the scores, the browser can jump to a complete set of solutions, and/or the background program can `ftp` the results to the instructor. This last feature I have not really used—only contemplated.



*Figures and illustrations* are handled by icons. Click on them and a graphics file pops up in separate window. FIGURE 1 Figure 1 represents the graph of the function  $f(x) = 3x^2 + 2x + 2$  and the line  $y = 8x - 2$ . The line is tangent to the function at the point  $(1, 6)$  as discussed in the quizzes above.

## Solutions to Exercises

**1.1.** You no doubt have realized that the solution to **EXAMPLE 1.1**, is, in fact, the solution to this exercise. The required area is,

$$A = \int_0^{\pi} \sin(x) dx = 2.$$

*Exercise Notes:* The values represents the area under one ‘hump’ of the sine graph. I bet you have always wondered how much area was under it. ■

Exercise 1.1. ■

# Solutions to Examples

**1.1.** We use standard techniques:

$$\begin{aligned}\int_0^\pi \sin(x) dx &= -\cos(x)|_0^\pi \\ &= \cos(x)|_\pi^0 \\ &= \cos(0) - \cos(\pi) \\ &= 1 - (-1) \\ &= 2\end{aligned}$$

*Presentation of Answer:*

$$\int_0^\pi \sin(x) dx = 2.$$

Example 1.1. ■

# Important Points

**That's Right!** We begin by calculating the *point of tangency*. The function is  $f(x) = 3x^2 + 2x + 1$  and we are interested in the case of  $x = 1$ ; therefore,  $f(1) = 6$ .

$$\textit{Point of Tangency: } (1, 6) \tag{I-1}$$

The next step is to find the *slope of the tangent line*.

$$f(x) = 3x^2 + 2x + 1 \implies f'(x) = 6x + 2$$

Evaluating the first derivative at our point of interest,  $x = 1$ , we obtain  $f'(1) = 8$ .

$$\textit{Slope of tangent line: } m_{\text{tan}} = f'(1) = 8. \tag{I-2}$$

Finally, we construct the equation of the tangent line:

$$y - y_0 = m(x - x_0) \quad \triangleleft \text{point-slope form}$$

$$y - 6 = 8(x - 1) \quad \triangleleft \text{(I-1) \& (I-2)}$$

$$y = 8x - 2 \quad \triangleleft \text{algebraic simplification}$$

*Presentation of answer:*  $y = 8x - 2.$

Important Point ■