THE UNIVERSITY OF AKRON
Mathematics and Computer Science

Learning and Exploring Mathematics

Last Revision Date: 7/20/1998

Essays

- How to Use this Tutorial.
- The Key to Success? It starts with an 'A'.
- Math Tests: Nothing to Fear!

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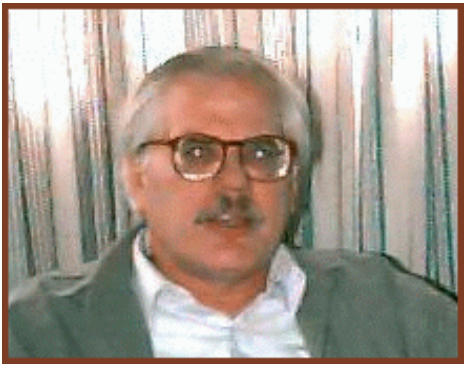

Photo by Mark Hamilton

Thanks are extended to the hard-disk of my monochrome screened IBM AT. By a lucky fluke of fate, by an act of shear fortune, by a *Poisson event* of the very rarest kind ... it crashed! Had it not crashed, I would not have gotten my current computer and this project would not even have been conceived.

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Acknowledgements

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Introduction

Patience
Dedication
Concentration
Supreme Concern

These algebra tutorials were written for those students who comtemplate coming to **The University of Akron**, though students and educators at other universities would certainly benefit as well. In the paragraphs that follow, I will try to outline the goals and the potential uses for these lessons.

For Students at The University of Akron. All students are required to take a **Math Placement Exam**. Each student will take one of two exams depending on their ACT Math or SAT Math Score. These algebra tutorials are aimed at preparing the student for the **Math Placement Exam II**. Students who do well on this exam will be placed in **Calculus I**; otherwise, they will be placed in **College Algebra** or **Precalculus**.

Many students entering **The University of Akron** do not know they must take a placement test; consequently, they are not properly

prepared to take it. (Some students have not taken a math course in high school as long ago as a year prior to matriculation.) As a result of this non-preparation, students do not perform up to their capabilities. This results in placing them in a lower level math course initially which, in turn, delays entry into the **Calculus** sequence.

These tutorials were designed to review the basics of high school algebra in hopes that students will use it to prepare for the placement exam. Working through these tutorials *may* lead to a higher score and a faster track toward major goals—**I offer no guarantees**, however. 𝔓𝔖

Students are allowed to retake the math placement exam after a waiting period of two weeks. Therefore, these tutorials were designed for a two week time interval.

Students may read all lessons or only those lessons needed to shore up deficient knowledge.

I want to emphasize that these tutorials are not meant to *teach you* algebra; they were written as a *review* of what you (theoretically)

already know! These tutorials will remind you of facts, ideas, and techniques you have already seen; they are meant to *freshen* your knowledge.

Students and Educators at other Universities. Many universities have placement exams similar to the ones at **The University of Akron**; therefore, these tutorials may represent an adequate review for those placement exams.

Students who need a general review in algebra—even those students already in **Calculus**—can utilize these lessons to improve their understanding.

Professional educators may provide this link to their students who need to review the basic concepts of algebra.

Finally, it is hoped these tutorials will be of value and provide a needed service to the general educational community. Nuff' said. 𝔐𝔓

How to Use this Tutorial

My advice: **Read, read, read, read, and, of course, read!**

You are reviewing algebra yourself; there is no instructor to lecture to you. You have to do it all! (Though I've done quite a bit for you already.) You must read and work through the lessons yourself.

The 10 lessons are designed for a two week interval. Hopefully, each is short enough that it can be read and worked through in a single day. There is not a large number of exercises, but there is enough.

Work all 10 lessons, or do only the ones you feel you need. Buy a notebook and in it write out all the sample examples. Study them carefully. Within your notebook, work the exercises—the answer/solutions can be found by clicking on the green EXERCISE. Important: Pay attention to and use correct notation, develop a nice style of presenting and displaying your algebraic thoughts; be organized, be neat, justify each and every step in much the same way as I do in the tutorials.

Your knowledge of and your ability to do algebra is key to your success in **Calculus** and beyond. Learn algebra well, and you will do well in all mathematics courses that follow.

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1. Features of this Document Delivery System

For the casual browser, that's you, here is a quick tour of the some basic features of these tutorials. The disk based system has more capabilities than the web based system. Below are features common to each system.

1.1. In-line Examples and Exercises

You will find throughout the tutorials, numerous examples and exercises mixed in with the discussions.

Examples are stated but the solutions are displayed only if the student wants to see them.

EXAMPLE 1.1. Solve for x in the equation $(x-1)^2 = x^2$.

Questions for the students are handled in the same way—through in-line exercises.

EXERCISE 1.1. Combine $\dfrac{1}{4x} - \dfrac{4}{x^3y} + \dfrac{3x}{2y^4}$.

1.2. Interactivity

Throughout the tutorials, there are multiple choice questions. There are several variations that are possible.

Below is an *immediate response question.*

Quiz. Is x^2 a factor of a factor of each term of $4x^3 - 9x^2 + x$?

(a) Yes (b) No

End Quiz.

Another type of question is immediate response *with solution.*

Quiz. Consider the rational expression: $\dfrac{x^4 + 3x^2}{x(x - 5x^3)}$. Is x^2 a *factor* of both numerator and denominator?

(a) Yes (b) No

End Quiz.

In the disk-based system, responses are all accompanied by annoying sounds.

In the disk-based system, it is possible to have graded quizzes. A background program keeps track of student responses and reports the score after the student has exited the quiz. After reporting the scores, the browser can jump to a complete set of solutions, and/or the background program can `ftp` the results to the instructor. This last feature I have not really used—only contemplated.

Figures and illustrations are handled by icons. Click on them and a graphics file pops up in separate window. FIGURE 1 Figure 1 represents the graph of the function $f(x) = 3x^2 + 2x + 2$ and the line $y = 8x - 2$. The line is tangent to the function at the point $(1, 6)$.

Solutions to Exercises

1.1. To find the LCD of

$$\frac{1}{4x} - \frac{4}{x^3y} + \frac{3x}{2y^4}$$

Follow the LCD "algorithm."

Step 1: **Factor each denominator completely.** In this example, this step is easy, though *Step* 1 has the potential of being the most difficult step.

First term has denominator: $4x$	Factors: 2^2, x
Second term has denominator: x^3y	Factors: x^3, y
Third term has denominator: $2y^4$	Factors: 2, y^4

Step 2: **Write down all factors.** The factors are

$$2^2, x, x^3, y, 2, y^4$$

Step 3: **Remove lowest powers.** Let's rearrange the previous display:

$$\underbrace{2,\, 2^2,}_{\text{same base}} \quad \underbrace{x,\, x^3,}_{\text{same base}} \quad \underbrace{y,\, y^4}_{\text{same base}}$$

Within each group, we eliminate all but the one with the highest exponent to obtain

$$2^2,\, x^3,\, y^4$$

Step 4: **Multiply all the factors together left after step 3 to obtain the least common denominator.** According to my LCD "algorithm," the least common denominator is

$$\text{LCD} = 2^2x^3y^4 = 4x^3y^4.$$

Now that we have jumped through hoops to get the LCD, we still have the job of combining the expression.

Now combine!

$$\begin{aligned}\frac{1}{4x} - \frac{4}{x^3y} + \frac{3x}{2y^4} &= \frac{1}{4x}\left[\frac{x^2y^4}{x^2y^4}\right] - \frac{4}{x^3y}\left[\frac{4y^3}{4y^3}\right] + \frac{3x}{2y^4}\left[\frac{2x^3}{2x^3}\right] \\ &= \frac{x^2y^4}{4x^3y^4} - \frac{16y^3}{4x^3y^4} + \frac{6x^4}{4x^3y^4} \\ &= \frac{x^2y^4 - 16y^3 + 6x^4}{4x^3y^4}\end{aligned}$$

Presentation of Answer:

$$\boxed{\frac{1}{4x} - \frac{4}{x^3y} + \frac{3x}{2y^4} = \frac{x^2y^4 - 16y^3 + 6x^4}{4x^3y^4}}$$

That was easy!

Once you understand and understand the LCD Algorithm, the process of getting the least common denominator will be ... second nature.

Exercise 1.1. ■

Solutions to Examples

1.1. Solve for x: $(x-1)^2 = x^2$.
Solution:

$$
\begin{aligned}
(x-1)^2 &= x^2 && \triangleleft \text{ given} \\
(x-1)^2 - x^2 &= 0 && \triangleleft \text{ subtract } x^2 \text{ from both sides} \\
[(x-1)-x][(x-1)+x] &= 0 && \triangleleft \text{ difference of squares!} \\
-(2x-1) &= 0 && \triangleleft \text{ combine} \\
2x-1 &= 0 && \triangleleft \text{ multipy by } -1 \\
2x &= 1 && \triangleleft \text{ add 1 to both sides} \\
x &= \frac{1}{2} && \triangleleft \text{ divide by 2}
\end{aligned}
$$

Comments: Note the *recommended* way of handling an equation with squares on both sides. Rather than taking the square root of both sides, take everything to one side of the equation and factor it as a *difference of squares*. *This is a much nicer method.* Example 1.1. ▪

Important Points

That's Right! Observe that

$$\frac{x^4 + 3x^2}{x(x - 5x^3)} = \frac{x^2(x^2 + 3)}{x^2(1 - 5x^2)}$$

That being the case, we would be honor bound to *cancel* this common factor to write

$$\frac{x^4 + 3x^2}{x(x - 5x^3)} = \frac{x^2 + 3}{1 - 5x^2}.$$

Important Point ■

Not Now, Silly!

Click on double left arrow on toolbar to return!