Calculus I | Assignment #3 | Due 4/21/97
Spring 1997 | Dr. D. P. Story

Homework to be handed in 4/21/97. This is the much anticipated assignment to be handed in. These are *your* problems—do them yourself! The assignment should be well-organized, well-written, and substantially correct. Show all details. Do a good job!

Have you ever wondered how much area in under one "hump" of the sine graph; perhaps we can't answer that question now, but at least we can move towards approximaing the area. Throughout this assignment, $f(x) = \sin(x)$, $0 \leq x \leq \pi$, is our function of interest—restricted to an interval of interest.

Problem. (Click here for the Solution) Consider the function $f(x) = \sin(x)$, $0 \leq x \leq \pi$, estimate the total area under the graph of f. In

the process of answering the questions below, make-up and fill-in a table of data like the one given here:

i	Δx_i	x_i	x_i^*	$f(x_i^*)$	$f(x_i^*)\,\Delta x_i$
0	⋮	⋮	⋮	⋮	⋮
1	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮
10	⋮	⋮	⋮	⋮	⋮
					R_{10}

1. Subdivide the interval $[\,0, \pi\,]$ into $n = 10$ subintervals all of the same length.

$$0 = x_0 < x_1 < x_2 < \cdots < x_9 < x_{10} = \pi \tag{1}$$

Calculate these *partition points* and list them in your table in the column labeled x_i.

2. Calculate the width of each of the subintervals and enter these values in your table in the column labeled Δx_i.

3. Choose a point, x_i^* from each of the 10 intervals: Students whose last names begin with A—M, choose the left-hand endpoint; students whose last names begin with N—Z, choose the right-hand endpoint. Put these values in your table.

4. Calculate the values $f(x_i^*) = \sin(x_i^*)$ for each of the values in the column labeled x_i^*, and enter these numbers in the column labeled $f(x_i^*)$.

5. Calculate the values in the column labeled $f(x_i^*)\,\Delta x_i$, and put them in this column. Write the entries using the first 6 significant digits. (And, once again, be sure to have your calculator in *radian mode.*

6. Calculate the column total of the column labeled $f(x_i^*)\,\Delta x_i$—this number is denoted by R_{10}, which stands for the 10^{th} Riemann Sum. The number R_{10} is the sum of the areas of your 10 rectangles. Enter this number in your table as shown above.

Conclusions. Summarize your findings by stating that

$$A = \int_0^{\pi} \sin(x)\, dx \approx R_{10},$$

where you replace the symbol R_{10} with your calculated value.