Calculus I | Assignment #1 | Due 2/17/97
Spring 1997 | Dr. D. P. Story

Homework to be handed in 2/17/97. This is the long awaited assignment to be handed in. These are *your* problems—do them yourself! The assignment should be well-organized, well-written, and substantially correct. Show all details. Do a good job!

1. (Calculator) The slope of the tangent line to the graph of the exponential function $y = 3^x$ at the point $(0, 1)$ is $\lim_{x\to 0}(3^x - 1)/x$. Estimate the slope to three decimal places by first making a table, similar to the ones on pages 52-53, and filling it in for values of $x = 0.5$, 0.1, 0.05, 0.01, 0.001, 0.0005, and 0.0001. **Solution.**

2. Estimate the value of
$$\lim_{x\to 0} \frac{6^x - 2^x}{x} \tag{1}$$
by graphing the function $y = (6^x - 2^x)/x$. State your answer to two decimal places. *Don't simply present the answer, explain your methodology.* (Note: If you do not have a graphing calculator, then

make up a table of values as in Problem #1.) **Can you guess the exact answer?** Let y_{est} be your estimated value for the limit in (1). Evaluate the various function buttons on your calculator at y_{est}; by doing so, can you give a reasonable guess as to the *exact value* of the limit in (1)? **Solution.**

3. Use a graph to find a number δ such that

$$|\sin(x) - \frac{1}{2}| < 0.5 \qquad \text{whenever} \quad |x - \frac{\pi}{6}| < \delta.$$

This can be down by graphing calculator (the easiest way) or a regular calculator. This problem is a nastier version of say problem #3, page 78 in the text. Whether you use a graphing calculator or not, document your methodology. **Solution.**

Problem 1. Approximate $\lim_{x \to 0} \dfrac{3^x - 1}{x}$

x	$f(x) = (3^x - 1)/x$	
.5	1.4641016	
.1	1.1612317	
.05	1.1293461	
.01	1.1046691	
.001	1.0992159	
.0005	1.0989140	
.0001	1.0986726	$\Longleftarrow$ This seems good!

It would appear that $\lim_{x \to 0} \dfrac{3^x - 1}{x} \approx 1.0986726$. Can we say that the *exact limit* is 1?

Problem 2. Approximate $\lim_{x\to 0} \dfrac{6^x - 2^x}{x}$.

Figure 1

Graphical Methods: Let's graph the function $f(x) = (6^x - 2^x)/x$ using several different scales. Consider FIGURE 1, what does the limit appear to be? The graph on the left is my first attempt, looks like the limit is a between 1 and 1.2. The graph on the right is the same graph with greater "zoom." Now it appears the limit is

$$\lim_{x\to 0} \frac{6^x - 2^x}{x} \approx 1.0984.$$

Numerical Calculation are no the next page.

Numerical Methods.

x	$f(x) = (6^x - 2^x)/x$	
.5	2.070552362	
.1	1.244577360	
.05	1.169172480	
.01	1.112352800	
.001	1.099979000	
.0005	1.099294000	
.0001	1.098750000	$\Longleftarrow$ This seems just as good!

It would appear that $\lim_{x\to 0} \dfrac{6^x - 2^x}{x} \approx 1.098750000$. Can we say that the *exact limit* is 1?

On your calculator, notice that $e^{1.098750000} \approx 3.000413162$. From this we infer that

$$1.098750000 \approx \ln(3.000413162).$$

Are you willing to say $\lim_{x\to 0} \dfrac{6^x - 2^x}{x} = \ln(3)$?

Problem 3. Calculate δ.

This problem is trivial. (No calculator even necessary.) The inequality $|\sin(x) - \frac{1}{2}| < 0.5$ becomes

$$0 < \sin(x) < 1$$

Certainly any value of x, $0 < x < \pi/2$, satisfies this inequality. Therefore, choose

$$\begin{aligned} \delta &= \min\left\{\frac{\pi}{6}, \frac{\pi}{2} - \frac{\pi}{6}\right\} \\ &= \min\left\{\frac{\pi}{6}, \frac{\pi}{3}\right\} \\ &= \frac{\pi}{6} \end{aligned}$$

Thus we take

$$\boxed{\delta = \frac{\pi}{6}.}$$