toc

9. Presentation of the Theory

In this section I have accumulated all theorems whose proofs had hypertext links.

Theorem 9.1. (Local Version) *Let f and g be functions and let a and c be number. Suppose f and g are continuous at $x = a$. Then*

(1) *the function $f + g$ is continuous at $x = a$;*
(2) *the function cf is continuous at $x = a$;*
(3) *the function fg is continuous at $x = a$;*
(4) *the function $\dfrac{f}{g}$ is continuous at $x = a$, provided, $g(a) \neq 0$.*

Proof: Under Construction.

Corollary 9.2. (Global Version) *Let f and g be functions that are continuous on a common domain A, and let c be a constant. Then each of the functions are continuous on the domain A: $f+g$, cf, and fg. In the case of the quotient function, f/g is continuous on the domain $B = \{\, x \in A \mid g(x) \neq 0 \,\}$.*

Proof: Under Construction.

Theorem 9.3. (The Composite Limit Theorem) *Let f and g be functions that are compatible for composition, let $a \in \mathbb{R}$. Suppose,*

(1) $\lim_{x\to a} g(x)$ *exists, let* $b = \lim_{x\to a} g(x)$*;*
(2) f *is continuous at* $b \in \operatorname{Dom}(f)$*.*

Then

$$\lim_{x\to a} f(g(x)), \ \textit{exists}$$

and,

$$\lim_{x\to a} f(g(x)) = f(b),$$

or,

$$\boxed{\lim_{x \to a} f(g(x)) = f(\lim_{x \to a} g(x)).} \qquad (1)$$

Proof: Under Construction.

Corollary 9.4. (Composition of Continuous Functions) *Let f and g be functions that are compatible for composition, let $a \in \mathbb{R}$. Suppose, g is continuous at $x = a$, and f is continuous at $g(a)$. Then $f \circ g$ is continuous at $x = a$; that is,*

$$\lim_{x \to a} f(g(x)) = f(g(a))$$

Proof: Under Construction.

Theorem 9.5. (The Intermediate Value Theorem) *Let f be a function defined on a closed interval $[\,a, b\,]$. Suppose M is a number strictly between $f(a)$ and $f(b)$. Then there is a number $c \in (\,a, b\,)$, such that $f(c) = M$.*

Proof: Under Construction.