

**Instructions:** (10 points) Solve each of the following problems without error. *Show all details.* Box in your answers. Use good notation, you *will* be marked off for bad notation.

- (4pts) 1. *Set up* the triple integral, in the rectangular coordinate system, for the volume of the solid bounded by the cylinder  $x = y^2$  and the planes  $z = 0$  and  $x + z = 1$ .

*Solution:* The set of all possible arrangements of the order of integration.

$$\begin{aligned} V &= \int_0^1 \int_{-\sqrt{x}}^{-\sqrt{x}} \int_0^{1-x} dz dy dx = \int_{-1}^1 \int_{y^2}^1 \int_0^{1-x} dz dx dy \\ &= \int_0^1 \int_0^{1-z} \int_{-\sqrt{x}}^{-\sqrt{x}} dy dx dz = \int_0^1 \int_0^{1-x} \int_{-\sqrt{x}}^{-\sqrt{x}} dy dz dx \\ &= \int_0^1 \int_{-\sqrt{1-z}}^{\sqrt{1-z}} \int_0^{1-z} dx dy dz = \int_{-1}^1 \int_0^{1-y^2} \int_0^{1-z} dx dz dy \end{aligned}$$

- (3pts) 2. (Problem #7, page 1074) *Set up* the integral  $\iiint_E \sqrt{x^2 + y^2} dV$  using the cylindrical coordinate system, where  $E$  is the solid inside the cylinder  $x^2 + y^2 = 16$  and between the planes  $z = -5$  and  $z = 4$ .

*Solution:* Converting to cylindrical, recalling  $r^2 = x^2 + y^2$  and that  $dV = r dz dr d\theta$ , we get

$$\iiint_E \sqrt{x^2 + y^2} dV = \int_0^{2\pi} \int_0^4 \int_{-4}^5 r^2 dz dr d\theta$$

- (3pts) 3. (Problem #17, page 1074) *Set up* the integral  $\iiint_B (x^2 + y^2 + z^2) dV$  using the spherical coordinate system, where  $B$  is the unit ball  $x^2 + y^2 + z^2 \leq 1$ .

*Solution:* Let's go spherical, shall we? Recall  $\rho^2 = x^2 + y^2 + z^2$  we get

$$\iiint_B (x^2 + y^2 + z^2) dV = \int_0^\pi \int_0^{2\pi} \int_0^1 \rho^4 \sin(\phi) d\rho d\theta d\phi$$