Calculus III | Pretest #3 | Name: ____________________

Fall, 2005 | | Dr. D. P. Story

Global Instructions: Test #13 will be Friday, November 18, 2005. The test covers the following sections in the text: §§15.6–15.7; §§16.1–16.6. The following are a selection of problems from the material to be covered on the test. These problems *do not* represent the entirely of the types of problems that may appear on the test. Solve these problems, ideally, without reference to your text.

1. Do each of the following.

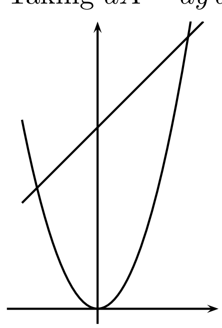
(a) *Solve* the iterated integral: $\int_0^1 \int_{x^3}^{2x} xy\, dy\, dx$. (*Note:* Continue on on the back of page 1, as needed.)

(b) Sketch the region of integration on on the back of page 1. (Be sure to shade in the region, and label the boundary curves.)

2. Sketch the solid whose volume is given by the iterated integral: $V = \int_0^1 \int_0^{1-x} 1 - x - y\, dy\, dx$.

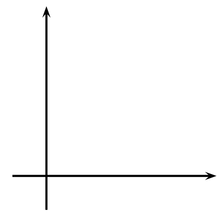
3. Consider the problem of calculating the integral $\iint_D f(x, y)\, dA$, where D is the region bounded by $y = x + 6$ and $y = x^2$.

(a) Taking $dA = dy\, dx$, *set up* the limits of integration;

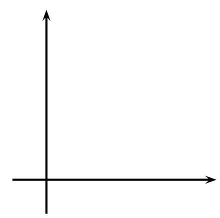


(b) Taking $dA = dx\,dy$, *set up* the limits of integration.

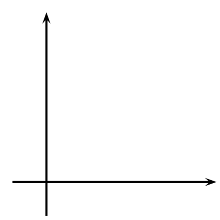
4. Reverse the order of integration: $\displaystyle\int_0^1 \int_{3y^2}^3 e^{x^2}\,dx\,dy$.



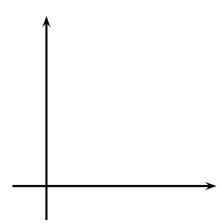
5. Consider the integral: $\displaystyle\iint_D \frac{1}{(1+x^2+y^2)^{3/2}}\,dA$. *Set up* the limits of integration for this integral by changing to polar coordinate variables. Here, D is the region in the first quadrant bounded by $y = x$, $x^2 + y^2 = 4$ and the y–axis.



6. *Set up* the double integral for the *volume* of the solid below the graph of the function $f(x, y) = x^2 + y^2$, and above the region D in the first quadrant of the xy–plane bounded by $x = 0$, $y = 0$, and $y = 4 - 2x$.



7. *Set up* the double integral for the moment about the y–axis for the region D in the first quadrant bounded by the parabola $y = x^2$ and the line $y = x$ given that the density function is $\rho(x, y) = x + y$.



8. Consider the following problem: Maximize/Minimize: $f(x, y, z) = x + 2y$
Subject to: $x + y + z = 1$
$x^2 + z^2 = 1$

Set up the Lagrange system of equations corresponding to this max/min problem.

9. Consider the problem of calculating the *surface area* of that portion of the graph of $f(x, y) = 9 - x^2 - y^2$ for (x, y) belonging to the region $D = \{ (x, y) \mid x^2 + y^2 \leq 9 \}$.

(a) *Set up* the integral necessary to solve this problem.

(b) Now, *calculate* the integral.

10. Consider the function $f(x, y) = x^3 - 3xy + 8y^3$. Find and classify all critical points.