Calculus III | Pretest #2 | Name: ____________________
Fall, 2005 | | Dr. D. P. Story

Global Instructions: Solve each of the following problems without error. Box in your answers. Be neat and organized.

1. The velocity of a particle at time t is given by $\vec{v}(t) = 2t^2\vec{\imath} - 3t\vec{\jmath}$.

(a) What is the *speed* of the particle at time $t = 2$?

speed =

(b) What is the *acceleration* of the particle at time $t = 2$?

accel =

2. Find the velocity and position vectors of a particle given the following information: $\vec{a}(t) = \vec{k}$, $\vec{v}(0) = \vec{\imath} - \vec{\jmath}$, and $\vec{r}(0) = \vec{0}$.

3. On back of page 1, make a sketch of the domain of the function: $f(x, y) = \dfrac{\ln(x^3 - y)}{\sqrt{x}}$.

4. Argue that $\displaystyle\lim_{(x,y)\to(0,0)} \frac{3xy}{4x^2 + 5y^2}$ does not exist.

5. Perform the indicated calculations.

(a) $f(x,y)=\sqrt{x^2y+4}$, calculate $f_x(-2,3)=$

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(b) $z=e^{xy}\sin(xy)$, calculate $\dfrac{\partial z}{\partial y}=$

$\dfrac{\partial z}{\partial y}=$

6. Let $w=f(x,y,z)$ and $\begin{cases} x=g(s,t) \\ y=h(s,t) \\ z=q(s,t) \end{cases}$. Use the *chain rule* to write a formula for $\partial z/\partial t$.

$$\frac{\partial w}{\partial t}=$$

7. Consider the function $f(x,y)=3x^4y^2$. Find the *equation* of the *plane tangent* to the graph of $f(x,y)$ at the point on the surface corresponding to $(x,y)=(1,-1)$.

8. Calculate $\dfrac{\partial z}{\partial x}$, where $3yz^2-x=y\sin(xz)$. (Continue on the back of page 1 as needed.)

$\dfrac{\partial z}{\partial x}=$

9. Let $f(x,y) = \dfrac{y}{x+y}$. It is easy to calculate $\nabla f(x,y) = \dfrac{\langle -y, x\rangle}{(x+y)^2}$. Using this fact, calculate $D_{\vec{u}} f(1,2)$, where $\vec{u}$ is the direction defined from $P(1,2)$ to $Q(4,6)$.

10. Consider the function $f(x,y) = x^3(2y-1)^2$.

(a) (4 pts) Calculate the maximum directional derivative of f at the point $(1,1)$.

(b) (3 pts) In what direction is the directional derivative at the point $(1,1)$ at its maximum? (Leave you answer as a unit vector $\vec{u}$.)

11. The volume of a cone of radius r and height h is $V = \frac{1}{3}\pi r^2 h$. The radius and height are measured to be $r = 1.3$ and $h = 4.1$, but the values are only accurate to $\pm.1$. Use differentials to estimate the error in the calculation of V.

12. Consider the equation $x^2z + z^2y^3 = z + 1$.

(a) Find the vector that is normal to the surface at $(1, 1, 1)$.

(b) Find the equation of the plane tangent to the surface $x^2z + z^2y^3 = z + 1$ at at $(1, 1, 1)$.

13. Let $f(x, y, z) = x^3y^4z^5 + xyz$. Verify that $f_{xxz} = f_{xzx}$.

(a) Calculation of f_{xxz}	(b) Calculation of f_{xzx}