Calculus III | Pretest #1 | Name: ______________
Fall 2005 | | Dr. D. P. Story

Instructions. Test #1 will be Monday, September 26, 2005. The test covers the following sections in the text: §§13.1-13.6; §§14.1,14.2. The following are a selection of problems from the material to be covered on the test. These problems *do not* represent the entirely of the types of problems that may appear on the test. Solve these problems, ideally, without reference to your text.

($3^{\text{pts}}_{\text{ea.}}$) **1.** Let $\vec{a} = \langle 2, -2, 1\rangle$, $\vec{b} = \langle 1, -2, 1\rangle$, $P(2, -3, 2)$, and $Q(-1, -2, 3)$. Calculate each of the following, show your work.

(a) $-3\vec{a} + 2\vec{b} =$

(b) $|\vec{b} - \vec{a}| =$

(c) the *direction cosines* of $\vec{a}$:

(d) $\overrightarrow{QP} =$ $\boxed{\langle \quad , \quad , \quad \rangle}$

(e) Let $\theta =$ the angle between $\vec{a}$ and $\vec{b}$. Then $\cos\theta =$

(f) Put the tail of $\vec{b}$ at the point Q, and calculate the coordinates of the head of the vector.

head $=$

(5^{pts}) **2.** Calculate the *area* of the *triangle* formed by putting the vectors $2\vec{\jmath}$ and $2\vec{\imath} - 3\vec{k}$ tail–to–tail.

area=

(8pts) **3.** Given the vectors $\vec{v} = \langle -1, 1, 3\rangle$ and $\vec{u} = \langle 2, -1, 2\rangle$, find a vector that is perpendicular to both $\vec{v}$ and $\vec{u}$.

(8pts) **4.** Consider $\vec{a} = \langle -2, 1, 3\rangle$ and $\vec{b} = \langle -1, 2, 1\rangle$. *Calculate* two vectors $\vec{\omega}_1$ and $\vec{\omega}_2$ such that $\vec{b} = \vec{\omega}_1 + \vec{\omega}_2$ where $\vec{\omega}_1 \parallel \vec{a}$ and $\vec{\omega}_2 \perp \vec{a}$.

$\vec{\omega}_1 = \langle \quad , \quad , \quad \rangle$	$\vec{\omega}_2 = \langle \quad , \quad , \quad \rangle$

(8pts) **5.** Write the parametric equations of the *line* passing through the point $P(-1, -2, 3)$ and perpendicular to the plane $-2x - 3y + 2z + 5 = 0$.

$$\begin{cases} x = \\ y = \\ z = \end{cases}$$

(8pts) **6.** Consider the plane $3x - 2y + 5z = 0$ and the line $x = -1 + 2t$, $y = 2 - t$, $z = 4t$. Find the point, P, where the line *intersects* the plane.

$P(\quad , \quad , \quad)$

(8^{pts}) **7.** Consider the two planes $x + y + x = 1$ and $x - 3y + z = 2$. *Calculate* the cosine of the acute angle these two planes make with each other.

$\cos\theta =$

(8^{pts}) **8.** Find the parametric equations that describe the *line tangent* to the graph of $\vec{r}(t) = (2t^2+1)\vec{i} - (1+e^{3t})\vec{j} + \sin 3t\vec{k}$ at the point on the graph corresponding to $t = 0$.

$$\begin{cases} x = \\ \\ y = \\ \\ z = \end{cases}$$

(8^{pts}) **9.** Let the position of a particle at time t is given by $\vec{r}(t) = \langle t^3, 5t^2, 6t\rangle$.

(a) Calculate $\vec{r}'(t)$

$\vec{r}'(t) = \langle \quad , \quad , \quad \rangle$

(b) Calculate $\vec{r}''(t)$

$\vec{r}''(t) = \langle \quad , \quad , \quad \rangle$

(c) Calculate $\vec{T}(t)$, the *unit tangent vector*, at $t = 1$.

$\vec{T}(1) = \langle \quad , \quad , \quad \rangle$

(8pts) **10.** Let $\vec{r}(t)$ satisfy the conditions $\vec{r}'(t) = 2e^{3t}\vec{i} + 3e^{-t}\vec{j}$ and $\vec{r}(0) = 3\vec{i}$. *Find* $\vec{r}(t)$.

$\vec{r}(t) = \langle \quad , \quad , \quad \rangle$

(8pts) **11.** Consider the hyperboloid $\dfrac{x^2}{4} - y^2 - \dfrac{z^2}{16} = 1$.

(a) Find the equation of the trace of this surface onto the xz–plane.

equation:

(b) Sketch the surface of the hyperboloid on back of on the back of page 3.

(5pts) **12.** Sketch the graph of the vector-valued function $\vec{r}(t) = \langle \cos t, t, \sin t \rangle$. [Do the sketch on on the back of page 3.]