Calculus III — Assignment #3 - Solution Notes — Date: 12/05/05
Fall 2005 — Dr. D. P. Story

1. Set up limits of integration. After carefully drawing the region and examining the various projections, we get:

   (a) $\displaystyle\int_0^{3/4}\int_x^{3(1-x)}\int_0^{(6-6x-2y)/3} f(x,y,z)\,dz\,dy\,dx$

   (b) $\displaystyle\int_0^{2}\int_0^{3(2-z)/8}\int_x^{(6-6x-3y)/2} f(x,y,z)\,dy\,dx\,dz$

   (c) $\displaystyle\int_0^{2}\int_0^{3(2-z)/8}\int_0^{y} f(x,y,z)\,dx\,dy\,dz + \int_0^{2}\int_{3(2-z)/8}^{(6-3y)/2}\int_0^{(6-2y-3z)/6} f(x,y,z)\,dx\,dy\,dz$

2. Consider the integral $\iint_R x-y\,dA_{xy}$, where the region R is the region bounded by the four lines $x+2y=0$, $x+2y=2$, $x-y=0$, and $x-y=3$. Evaluate the integral by using the transformation $u=x+2y$, $v=x-y$ to change the variables of integration.

   Solution: Solving for x and y we obtain:

   $$x=\tfrac{1}{3}(u+2v) \qquad y=\tfrac{1}{3}(u-v)$$

   The Jacobian can now be easily computed to be $|\partial(x,y)/\partial(u,v)| = |-\frac{1}{3}| = \frac{1}{3}$. The computation of the boundary curves is easy enough in the $uv-plane$. Thus,

   $$\iint_R x-y\,dA_{xy} = \int_0^2\int_0^3 v\,(\tfrac{1}{3})\,dv\,du = \boxed{3}$$

   The student, that's you, should verify details.