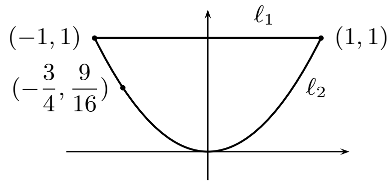
Calculus III	HW #2–Section 004	Name: ______________
Fall 2005		Dr. D. P. Story

(10pts) **1.** (Find Absolute Max/Min.) Consider the function $f(x,y)=3x+2y$ defined over the closed and bounded region $D=\{\,(x,y) : x^2\le y\le 1\,\}$. Find the absolute maximum and absolute minimum of the function f over the domain D using the techniques of §15.7.

Solution: The function $f(x,y)=3x+2y$ has gradient $\nabla f(x,y)=\langle 3,2\rangle$; obviously, the gradient can never be equal to the zero vector, so the maximum and minimum must occur on the boundary of the given domain

$$D=\{\,(x,y) : x^2\le y\le 1\,\}$$



Parameterize boundary line, ℓ_1,
$\ell_1 : \begin{cases} x=t \\ y=1 \end{cases}, -1\le t\le 1$. Now substitute into the function $f(x,y)=3x+2y$ to obtain

$$g(t)=f(t,1)=3t+2, -1\le t\le 1$$

This straight line has not critical point, but we must be sure to include the endpoints. Thus, $(-1,1)$ and $(1,1)$ are critical points.

Summary Analysis

(x,y)	$f(x,y)$	Conclusion
$(-1,1)$	-1	
$(1,1)$	5	abs. max.
$(-\frac{3}{4},\frac{9}{16})$	$-\frac{9}{8}$	abs. min.

Parameterize boundary curve, ℓ_2
$\ell_2 : \begin{cases} x=t \\ y=t^2 \end{cases}, -1\le t\le 1$. Now substitute into the function $f(x,y)=3x+2y$ to obtain

$$g(t)=f(t,t^2)=3t+2t^2, -1\le t\le 1$$

Now, differentiating this function with respect to t, we obtain $g'(x)=3+4t$. This function has a critical number at $t=-3/4$; hence, $(-3/4,9/16)$ is a critical point of our problem.

The calculation to the left indicated the locations and values of the absolute maximum and absolute minimum.

(10pts) **2.** Using Lagrange Multipliers (§15.8 in text), find the maximum and minimum of the function $f(x,y,z)=x-y+3z$ subject to the constraint $x^2+y^2+4z^2=1$.

Solution: We set up the Lagrange Multiplier system of equations: $f(x,y,z)=x-y+3z$ and $g(x,y,z)=x^2+y^2+4z^2=1$. We set up the equations $\nabla f=\lambda\nabla g$ to obtain:

$$\begin{aligned} 1 &= 2\lambda x & x &= \frac{1}{2\lambda} \\ -1 &= 2\lambda y & y &= -\frac{1}{2\lambda} \\ 3 &= 8\lambda z & z &= \frac{3}{8\lambda} \\ x^2+y^2+4z^2 &= 1 \end{aligned}$$

Now solve for x, y, and z in the first three equations and substitute into the fourth: $\left(\frac{1}{2\lambda}\right)^2+\left(-\frac{1}{2\lambda}\right)^2+4\left(\frac{3}{8\lambda}\right)^2=1$. Very carefully, we solve for λ, $\boxed{\lambda=\pm\sqrt{17}/4}$. The critical points can now be computed and the max and min identified:

(x,y)	$f(x,y)$	Conclusion
$(2/\sqrt{17},-2/\sqrt{17},3/(2\sqrt{17}))$	$\sqrt{17}/2$	abs. max
$(-2/\sqrt{17},2/\sqrt{17},-3/(2\sqrt{17}))$	$-\sqrt{17}/2$	abs. min