THE UNIVERSITY OF AKRON
Theoretical and Applied Mathematics

Equivalent Fractions & Reducing Fractions

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May 13, 2004

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1. Introduction

Being able to convert one fraction into an equivalent fraction is a very important skill that is used in adding and subtracting fractions, and in reducing fractions to the their lowest terms. This document presents short tutorials on two basic skills needed in grade school: writing equivalent fractions and reducing fractions.

2. Equivalent Fractions

Converting one fraction into an equivalent fraction is a very important skill. This skill plays a vital role in the problem of adding or subtracting two fractions when the denominators of the two fractions are different.

Adding or subtracting two fractions is easy when the denominators are the same, for example,

$$\frac{2}{5} + \frac{6}{5} = \frac{2+6}{5} = \boxed{\frac{8}{5}}$$

you simply add (or subtract) the numerators, as above. When the

denominators are different, such as

$$\frac{2}{3} + \frac{1}{6} \tag{1}$$

the problem is not solved quite so easily.

The strategy for adding or subtracting fractions with different denominators is to replace one or both fractions with equivalent fractions that all have the same denominator. In the above example, it is my idea to write the first fraction equivalent to a fraction with a denominator of 6, that way, all fractions will have the same denominator. So I ask myself the question

$$\frac{2}{3} = \frac{?}{6}$$

How do I figure out what the numerator should be? I get an answer of

$$\frac{2}{3} = \frac{4}{6} \tag{2}$$

Now, returning to problem of adding the two fractions given in (1),

we have

$$\frac{2}{3}+\frac{1}{6}=\frac{4}{6}+\frac{1}{6}=\frac{1+4}{6}=\boxed{\frac{5}{6}}$$

where I have replaced the fraction 2/3 with the equivalent fraction 4/6. Once this has been accomplished, the addition problem becomes easy.

In this section, we concentrate on the skill of converting one fraction into an equivalent fraction, this was the skill I used in (2) to obtain a fraction with a denominator of 6.

2.1. A Short Lesson

Now, let's discuss the strategy for writing equivalent fractions. There are two basic methods that we use:

1. We can *multiply* both numerator and denominator by the same number, and we will create a new fraction equivalent to the original one;
2. we can *divide* both numerator and denominator by the same number, and we will again create a new fraction equivalent to

the original one.

In this lesson, we will use method (1) to create equivalent fractions; in the section on Reducing Fractions, we'll use method (2).

Problem: Convert a fraction $\frac{a}{b}$ to an equivalent fraction having a specified denominator, d. That is, write

$$\frac{a}{b} = \frac{?}{d}$$

the problem is to figure out what the numerator is (the '?').

To solve this kind of problem, most likely we multiply numerator and denominator by some cleverly chosen number.

Example 1. Write the fraction $\frac{2}{3}$ with a denominator of 6, that is,

$$\frac{2}{3} = \frac{?}{6}$$

Solution: We ask ourself, 3 (the denominator we want to change) times what number is equal to 6 (the denominator we want to change

to). The answer is 2 since $3 \cdot 2 = 6$. So...

$$\frac{2}{3} = \frac{2 \cdot 2}{3 \cdot 2} = \boxed{\frac{4}{6}}$$

Rather than straining our brain looking for a number which multiplied by 3 gives 6, we can simply divide. How many times does 3 go into 6, that is $\frac{6}{3} = 2$ and 2 is the number we are looking for. □

Example 2. Write

$$\frac{3}{5} = \frac{?}{20}$$

Solution: We ask ourself, 5 (the denominator we want to change) times what number is equal to 20 (the denominator we want to change to). The answer is 4 since $5 \cdot 4 = 20$. So...

$$\frac{3}{5} = \frac{3 \cdot 4}{5 \cdot 4} = \boxed{\frac{12}{20}}$$

We could have computed $\frac{20}{5} = 4$ to get the 4 we need. □

2.2. The Method

Before trying to do some problems on your own, let's reduce our method down to some simple steps.

Problem: Write $\frac{5}{6} = \frac{?}{24}$.

1. Divide the denominator 6 into the denominator 24: $\frac{24}{6} = 4$.
2. Multiply the numerator of the left-hand numerator by the number, 4, just computed in Step (1), to get the correct numerator of the right-hand side:

$$\frac{5}{6} = \frac{5 \times 4}{24} = \frac{20}{24}$$

2.3. Some Quizzes

Test your understanding of the lesson by trying some of the quizzes that follow. Begin by clicking on the "Begin" button. Enter your answer where you see the question marks. When you are finished, click on "End" button to see your how you did on the quiz. Click on

the “Correct” button to get the answers. Click on the “Ans” button to see the correct answer; if this button has a green border, you can shift-click to see a more detailed solution of this problem. If you jump to a solutions, click on the green square to jump back to your quiz.

Begin Convert the given fraction into one with the specified denominator. Work in the margins or on scratch paper.

1. $\frac{2}{3} = \frac{?}{15}$ ☐

2. $\frac{4}{5} = \frac{?}{25}$ ☐

3. $\frac{1}{4} = \frac{?}{24}$ ☐

4. $\frac{7}{9} = \frac{?}{18}$ ☐

5. $\frac{3}{10} = \frac{?}{40}$ ☐

End Score: Correct Answers: _____

Begin Convert the given fraction into one with the specified denominator. Work in the margins or on scratch paper.

1. $\frac{3}{8} = \frac{?}{24}$ ☐

2. $\frac{3}{7} = \frac{?}{28}$ ☐

3. $\frac{2}{6} = \frac{?}{54}$ ☐

4. $\frac{6}{12} = \frac{?}{48}$ ☐

5. $\frac{3}{4} = \frac{?}{16}$ ☐

End Score: Correct Answers: _____

Another, less important skill, is to write a fraction with a given numerator equivalent to a given one. The method of solution is the same, except we work with the numerator rather than the denominator. Try these and see how you do.

Begin Convert each fraction to the indicated equivalent fraction.

1. $\frac{1}{2} = \frac{12}{?}$ ☐

2. $\frac{3}{4} = \frac{15}{?}$ ☐

3. $\frac{2}{3} = \frac{14}{?}$ ☐

4. $\frac{2}{5} = \frac{6}{?}$ ☐

5. $\frac{6}{5} = \frac{24}{?}$ ☐

End Score: Correct Answers: _____

3. Reducing Fractions

When presenting your final answer to an arithmetic problem, it is important to reduce your answer to lowest terms. The process of reducing a fraction to lowest terms is similar to converting a fraction to an equivalent fractions; usually, we divide rather than multiply both numerator and denominator by the same number to get the reduction.

We **reduce** a fraction by writing an equivalent fraction with a *smaller denominator*. This is done by dividing the numerator and denominator by the same number. For example,

$$\frac{8}{12} = \frac{8/2}{12/2} = \frac{4}{6} \tag{3}$$

Here, we have reduced 8/12, which has a denominator of 12, to an equivalent fraction, 4/6, which has a denominator of 6. The denominator has been reduced (in size).

Example 1. The fraction $\frac{2}{3}$ is a reduced form of $\frac{4}{6}$ since $\frac{4}{6} = \frac{2}{3}$ *and* the denominator of $\frac{2}{3}$ smaller than the denominator of $\frac{4}{6}$.

▶ A fraction a/b is reduced to lowest terms if it cannot be reduced.

In the case of Example 1, we reduced $\frac{4}{6}$ to $\frac{2}{3}$. This fraction, $\frac{2}{3}$, is reduced to lowest terms, it cannot itself be reduced further.

Example 2. The fraction $\frac{12}{18}$ can be reduced to $\frac{6}{9}$, that is, $\frac{12}{18} = \frac{6}{9}$, but $\frac{6}{9}$ itself can be reduced:

$$\frac{12}{18} = \frac{6}{9} = \frac{2}{3}$$

The reduced fraction $\frac{2}{3}$ is reduced to lowest terms. This is the best answer.

Now, how do we reduced fractions? Reduction is based on a simple property of arithmetic:

1. If we *multiply* the numerator and denominator by the same number, we do not change the value of the fraction.
2. If we *divide* the numerator and denominator by the same number, we do not change the value of the fraction.

To reduce fractions, we use (2) above: divide numerator and denominator by the same number.

Example 3. Express (reduce) the fraction $\frac{9}{21}$ in lowest terms.

Solution: The strategy is to divide a the numerator and denominator by the same number. Look at the given fractions $\frac{9}{21}$. We must think of a number that divides both the numerator and denominator. (At our level of play, look at some common values: 2, 3, 5)

After a few moments of meditation, we see that 3 divides numerator and denominator of the given fraction. Thus,

$$\frac{9}{21} = \frac{9/3}{21/3} = \boxed{\frac{3}{7}} \tag{4}$$

To see if this is in lowest terms, we try again to find a number that divides the numerator and denominator of our new fraction $\frac{3}{7}$. We see there is number that divides both numerator and denominator. So, or, or answer in (4) is reduced. □

Remember: Look at the given fraction, and try to think of a number that divides both numerator and denominator. Once that is found,

divide!

Example 4. Express (reduce) the fraction $\frac{15}{60}$ in lowest terms.

Solution: The strategy is to divide a the numerator and denominator by the same number. Look at the given fractions $\frac{15}{60}$. We must think of a number that divides both the numerator and denominator. (At our level of play, look at some common values: 2, 3, 5)

The number 5 is an obvious choice,

$$\frac{15}{60} = \frac{15/5}{60/5} = \frac{3}{12}$$

Is this expressed in lowest terms? No! We see that 3 divides both numerator and denominator, so...

$$\frac{15}{60} = \frac{15/5}{60/5} = \frac{3}{12} = \frac{3/3}{12/3} = \boxed{\frac{1}{4}} \tag{5}$$

□

Begin Reduce each of the following fractions to lowest term.

1. $\frac{4}{6} =$ —— ☐

2. $\frac{8}{16} =$ —— ☐

3. $\frac{8}{12} =$ —— ☐

4. $\frac{18}{24} =$ —— ☐

5. $\frac{8}{20} =$ —— ☐

End Score: Correct Answers: _____

That was so much fun, let's try more of the same!

Begin Reduce each of the following to lowest terms.

1. $\frac{24}{28} = \frac{\quad}{\quad}$ ☐

2. $\frac{3}{7} = \frac{\quad}{\quad}$ ☐

3. $\frac{15}{24} = \frac{\quad}{\quad}$ ☐

4. $\frac{10}{15} = \frac{\quad}{\quad}$ ☐

5. $\frac{8}{36} = \frac{\quad}{\quad}$ ☐

End Score: Correct Answers: _____

Need more practice?

Begin Reduce each of the following to lowest terms.

1. $\frac{56}{72} =$ —— ☐

2. $\frac{18}{48} =$ —— ☐

3. $\frac{32}{40} =$ —— ☐

End Score: Correct Answers: _____

All these problems are done the same way. Doing many problem reenforces the technique. When you see a problem of this type in the future, just apply these standard techniques!

Solutions to Quizzes

Solution to Quiz: We have $15/3 = 5$, so we multiply the numerator and denominator by 5.

$$\frac{2}{3} = \frac{2 \cdot 5}{3 \cdot 5} = \boxed{\frac{10}{15}}$$

Solution to Quiz: We have $25/5 = 5$, so multiply the numerator and denominator by 5.

$$\frac{4}{5} = \frac{4 \cdot 5}{5 \cdot 5} = \boxed{\frac{20}{25}}$$

Solution to Quiz: We have $24/4 = 6$, so we multiply the numerator and denominator by 6.

$$\frac{1}{4} = \frac{1 \cdot 6}{4 \cdot 6} = \boxed{\frac{6}{24}}$$

Solution to Quiz: We have $18/9 = 2$, so we multiply the numerator and denominator by 2.

$$\frac{7}{9} = \frac{7 \cdot 2}{9 \cdot 2} = \boxed{\frac{14}{18}}$$

Solution to Quiz: We have $40/10 = 4$, so we multiply the numerator and denominator by 4.

$$\frac{3}{10} = \frac{3 \cdot 4}{10 \cdot 4} = \boxed{\frac{12}{40}}$$

Solution to Quiz: We have $24/8 = 3$, so we multiply the numerator and denominator by 3.

$$\frac{3}{8} = \frac{3 \cdot 3}{8 \cdot 3} = \boxed{\frac{9}{24}}$$

Solution to Quiz: We have $28/7 = 4$, so we multiply the numerator and denominator by 4.

$$\frac{3}{7} = \frac{3 \cdot 4}{7 \cdot 4} = \boxed{\frac{21}{28}}$$

Solution to Quiz: We have $54/6 = 9$ so we multiply the numerator and denominator by 9.

$$\frac{2}{6} = \frac{2 \cdot 9}{6 \cdot 9} = \boxed{\frac{18}{54}}$$

Solution to Quiz: We have $48/12 = 4$ so we multiply the numerator and denominator by 4.

$$\frac{6}{12} = \frac{6 \cdot 4}{12 \cdot 4} = \boxed{\frac{24}{48}}$$

Solution to Quiz: We have $16/4 = 4$ so we multiply the numerator and denominator by 4.

$$\frac{3}{4} = \frac{3 \cdot 4}{4 \cdot 4} = \boxed{\frac{12}{16}}$$

Solution to Quiz: Divide numerator and denominator by 8.

$$\frac{8}{16} = \frac{8/8}{16/8} = \boxed{\frac{1}{2}}$$

You could have also reduced in stages, first by 2, then 2 again, then 2 a third. There are several other possible ways of reducing this fraction, can you name one other way? ■

Solution to Quiz: Divide numerator and denominator by 4.

$$\frac{8}{12} = \frac{8/4}{12/4} = \boxed{\frac{2}{3}}$$

Solution to Quiz: Divide numerator and denominator by 6.

$$\frac{18}{24} = \frac{18/6}{24/6} = \boxed{\frac{3}{4}}$$

You could have also reduced in stages, first by 2, then by 3. ■

Solution to Quiz: Divide numerator and denominator by 4.

$$\frac{8}{20} = \frac{8/4}{20/4} = \boxed{\frac{2}{5}}$$

Solution to Quiz: Divide numerator and denominator by 4.

$$\frac{24}{28} = \frac{24/4}{28/4} = \boxed{\frac{6}{7}}$$

Solution to Quiz: This fraction is already reduced to lowest terms, there is no number that divides both 3 and 7. The answer is

$$\boxed{\frac{3}{7}}$$

Solution to Quiz: Divide numerator and denominator by 3.

$$\frac{15}{24} = \frac{15/3}{24/3} = \boxed{\frac{5}{8}}$$

Solution to Quiz: Divide numerator and denominator by 5.

$$\frac{10}{15} = \frac{10/5}{15/5} = \boxed{\frac{2}{3}}$$

Solution to Quiz: Divide numerator and denominator by 4.

$$\frac{8}{36} = \frac{8/4}{36/4} = \boxed{\frac{2}{9}}$$

You could have also reduced in stages, first by 2, then by 3. ■

Solution to Quiz: Divide numerator and denominator by 8.

$$\frac{56}{72} = \frac{56/8}{72/8} = \boxed{\frac{7}{9}}$$

It may not have been obvious to divide by 8, but it would be obvious to divide by 2

$$\frac{56}{72} = \frac{56/2}{72/2} = \frac{28}{36}$$

now by 2 again!

$$= \frac{28/2}{36/2} = \frac{14}{18}$$

divide one more time by 2!!

$$= \frac{14/2}{18/2} = \boxed{\frac{7}{9}}$$

What we finally end up dividing by? We divided by 2, then 2 again,

finally by 2 a third time. In total, we divided by $2 \cdot 2 \cdot 2 = 8$, which is what we divided by in our first solution. ■

Solution to Quiz: Divide numerator and denominator by 6.

$$\frac{18}{48} = \frac{18/6}{48/6} = \boxed{\frac{3}{8}}$$

Solution to Quiz: Divide numerator and denominator by 8.

$$\frac{32}{40} = \frac{32/8}{40/8} = \boxed{\frac{4}{5}}$$