Test Total

Name ______________________________

Sample Final S'08 Linear Algebra 3450:312 Dr. Clemons
Show your work.

1. Let $A = \begin{bmatrix} 1 & -3 & 5 \\ 0 & 1 & 0 \\ 0 & -2 & 2 \end{bmatrix}$.

(a) Show that the characteristic equation is $(1-\lambda)^2(2-\lambda)$.

10 pts

(b) Eigenvalue $\lambda = 2$ has eigenvector $\begin{pmatrix} 5 \\ 0 \\ 1 \end{pmatrix}$. Find a basis for the eigenspace of $\lambda = 1$.

10 pts

(c) Is matrix A diagonalizable? If so, write a matrix P and diagonal matrix D such that $P^{-1}AP = D$.

5 pts

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2. Find the least squares solution to $A\mathbf{x} = \mathbf{b}$, where $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 0 \end{pmatrix}$, and $\mathbf{b} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$.

10 points

3. Consider the transformation $T : \mathbf{R}^4 \to \mathbf{R}^2$, given by $T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 + 7x_2 + x_3 \\ x_1 + x_3 \end{pmatrix}$

(a) Show that T is a linear transformation.

8 pts

(b) Find the matrix representation of T with respect to the standard bases.

10 pts

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4. Determine whether or not the vectors 1, x^2, and $x^2 - 2$ are linearly independent in $\mathbf{P}_2$. Do these vectors form a basis for $\mathbf{P}_2$?

10 pts

5. Show that the null space for matrix $A_{m \times n}$ is a subspace? Is it a subspace of $\mathbf{R}_m$ or $\mathbf{R}_n$

7 pts

6. If λ is an eigenvalue of matrix A having corrsponding eigenvector $\mathbf{x}$, what is and eigenvalue and eigenvector for A^2.

6 pts

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7. Let W be the subspace of $\mathbf{R}^4$ spanned by the linearly independent vectors $\mathbf{v}_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$, $\mathbf{v}_2 = \begin{pmatrix} 3 \\ 1 \\ 1 \\ 3 \end{pmatrix}$, and $\mathbf{v}_3 = \begin{pmatrix} 5 \\ 0 \\ 2 \\ 3 \end{pmatrix}$. The Gram-Schmidt orthonormalization process has produced $\mathbf{u}_1 = \begin{pmatrix} 1/2 \\ 1/2 \\ 1/2 \\ 1/2 \end{pmatrix}$ and $\mathbf{u}_2 = \begin{pmatrix} 1/2 \\ -1/2 \\ -1/2 \\ 1/2 \end{pmatrix}$.

(a) Find $\mathbf{u}_3$

10 pts

(b) Given $\mathbf{y} = \begin{pmatrix} 2 \\ 0 \\ 0 \\ 2 \end{pmatrix}$, find the projection of $\mathbf{y}$ onto W.

10 pts

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8. Let $A = \begin{pmatrix} 1 & 1 \\ 0 & 1/\sqrt{2} \\ 0 & 1/\sqrt{2} \end{pmatrix}$, $Q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$, and $\mathbf{b} = \begin{pmatrix} 1 \\ \sqrt{2} \\ \sqrt{2} \end{pmatrix}$.

(a) Find an upper triangular matrix R such that $A = QR$.

10 pts

(b) Use the QR factorization to find the least-squares solution of $A\mathbf{x} = \mathbf{b}$.

10 pts

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9. Given that $T(1,0,1) = (2,3)$ and $T(0,1,1) = (4,-2)$,

6 pts

(a) Define T as a linear transformation from $\mathbf{R}^3$ to $\mathbf{R}^2$.

(b) Find $\mathbf{x}$ in $\mathbf{R}^3$ such that $T(\mathbf{x}) = (0,-8)$

10 pts

10. Let $A = \begin{pmatrix} 1 & 2 & 7 \\ -2 & 5 & 4 \\ -5 & 6 & -3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 3 \\ 3 \\ 1 \end{pmatrix}$.

(a) Write the solution set for the system $A\mathbf{x} = \mathbf{b}$.

10 pts

(b) What is the rank and nullity of A?

6 pts

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