

Test Total

Name _____

Sample Test 2 Linear Algebra 3450:312 Dr. Clemons

Show your work.

1. Let $A = \begin{bmatrix} 2 & 0 & -1 & 2 & -2 & -2 \\ 2 & 1 & -1 & 0 & -1 & 2 \\ 4 & 0 & -2 & 4 & -3 & -1 \\ 2 & 1 & -1 & 0 & -2 & -1 \end{bmatrix}$, which row-reduces to $\begin{bmatrix} 1 & 0 & -\frac{1}{2} & 1 & 0 & 2 \\ 0 & 1 & 0 & -2 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

(a) Find a basis for $Col A$.

5 points

(b) Find a basis for $Nul A$.

10 points

(c) Find a basis for $Row A$.

5 points

(d) Find $Rank A$ and $dim Nul A$.

5 points

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2. Use Cramer's rule to solve

$$\begin{aligned}2x_1 + x_2 - x_3 &= 4 \\x_1 + x_2 + 3x_3 &= 2 \\-x_2 + 5x_3 &= 1\end{aligned}$$

10 points

3. Let \mathbf{P}_2 be the space of polynomials of degree two or less. The sets $A = \{t - 1, t + 1, t^2\}$ and $B = \{2t, 2t + t^2, 2 - t^2\}$ form bases for \mathbf{P}_2 .

(a) Calculate the change-of-coordinate matrix $[P]_{A \leftarrow B}$.

10 points

(b) Evaluate $[2t + t^2]_A$ and check that $[P]_{A \leftarrow B}[2t + t^2]_B = [2t + t^2]_A$.

10 points

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4. Suppose that $\det A = 3$, and that B is obtained from A by interchanging row_1 and row_3 , and multiplying row_2 by 5.

Evaluate $\det AB$.

5 points

5. Define $H = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : ad - bc \geq 0 \right\}$.

Show that H is *not* a subspace of $M_{2 \times 2}$.

10 points

6. Consider $H = \{p(t) = c_0 + c_1t + c_2t^2 : p(2) = 0\}$, which is a subspace of π_2 , of dimension 2. Find 2 *linearly independent* vectors (polynomials) in H .

10 points

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7. Define the *linear transformation* $T : M_{2 \times 2} \rightarrow \pi_3$ by

$$T \left(\begin{bmatrix} a & b \\ c & d \end{bmatrix} \right) = (a + c) + (2a + b)t + (c + 2d)t^3.$$

Find a basis for *ker* T .

10 points

Find a basis for *Range* T .

10 points

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