Test Total

Name ______________________________

Sample Test 1 Linear Algebra 3450:312

Show your work.

1. Use row-reduction on the *augmented matrix* to find all solutions to the following linear system. Write your solution in *vector form.*

$$\begin{aligned} x_1 - 2x_2 + x_4 &= 2 \\ 2x_1 - 4x_2 + x_3 + x_4 &= 7 \\ 2x_1 - 4x_2 + 2x_4 &= 4 \end{aligned}$$

15 points

2. Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$. Find the LU-decomposition of A. Find A^{-1} using inverse formula for a 2×2 matrix.

15 points

3. Suppose that $\{\underline{u}_1, \underline{u}_2, \underline{u}_3, \underline{u}_4\}$ is *linearly dependent.*

Is it *necessarily* true that we can always write $\underline{u}_4$ as a linear combination of $\{\underline{u}_1, \underline{u}_2, \underline{u}_3\}$?

Carefully explain your answer.

10 points

4. Use row-reduction to find the inverse of $\begin{bmatrix} 1 & 1 & 2 \\ 1 & -1 & 3 \\ 2 & 2 & 3 \end{bmatrix}$.

15 points

5. Show that if $\{\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3\}$ is linearly dependent and $A_{3\times 3}$ is *one-to-one*, then $\{\mathbf{y}_1, \mathbf{y}_2, \mathbf{y}_3\}$ is linearly dependent, where $\mathbf{y}_1 = A\mathbf{x}_1$, $\mathbf{y}_2 = A\mathbf{x}_2$ and $\mathbf{y}_3 = A\mathbf{x}_3$.

10 points

6. Find the values of a, b, c so that the following matrix multiplication holds :

$$\begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 2 \\ 0 & -1 & 2 \\ 0 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 2 \\ -\frac{2}{3} & -\frac{4}{3} & \frac{4}{3} \\ 1 & \frac{5}{2} & 1 \end{bmatrix}$$

10 points

7. Suppose that $A \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -5 \end{bmatrix} = A \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $A\underline{x} = \underline{0}$ has exactly one free variable.

Find the general solution to $A\underline{x} = \underline{0}$.

10 points

8. Suppose that T is a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^4$, which satisfies

$T(2,1) = (1,3,5,7)$ and $T(4,7) = (-1,1,-1,1)$

Evaluate $T(3,5)$

10 points

9. A is a 3x3 matrix. To reduce it to the *identity matrix*, we first exchange row_1 and row_2, and then replace row_3 by $row_3 + row_2$.

What is A?

5 points