$\bar{X} = 86.8$
$n = 33$
$\sigma = 9$
min = 67
med = 87
max = 100

Test Total

5 - 100
7 - 90's
15 - 80's
4 - 70's
2 - 60's

Name KRY

Test 2 Linear Algebra 3450:312 Dr. Clemons
Show your work.

1. Let $A = \begin{bmatrix} 2 & -3 & -7 & -1 & 7 \\ 1 & 0 & -2 & 1 & 2 \\ 0 & -2 & -2 & -2 & 2 \\ 1 & 1 & -1 & -1 & -2 \end{bmatrix}$, which row-reduces to $\begin{bmatrix} 1 & 0 & -2 & 0 & 1 \\ 0 & 1 & 1 & 0 & -2 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

(a) Find a basis for *Col A*.

$\beta = \left\{ \begin{pmatrix} 2 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -3 \\ 0 \\ -2 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ -2 \\ -1 \end{pmatrix} \right\}$

5 points

(b) Find a basis for *Nul A*.

$x_1 = 2x_3 - x_5$
$x_2 = -x_3 + 2x_5$
$x_3 = x_3$
$x_4 = -x_5$
$x_5 \quad x_5$

$\beta = \left\{ \begin{pmatrix} 2 \\ -1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 2 \\ 0 \\ -1 \\ 1 \end{pmatrix} \right\}$

10 points

(c) Find a basis for *Row A*.

$\beta = \left\{ \begin{pmatrix} 2 \\ -3 \\ -7 \\ -1 \\ 7 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -2 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ -2 \\ -2 \\ -2 \\ 2 \end{pmatrix} \right\}$ or $\left\{ \begin{pmatrix} 1 \\ 0 \\ -2 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \\ -2 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \end{pmatrix} \right\}$

5 points

(d) Find *Rank A* and *dim Nul A*.

Rank + Null = # Col = 5

$3 + 2 = 5$

5 points

Page 1 Total (25)

2. Use Cramer's rule to solve

$$3x + 7y = 5$$
$$2x + 4y = 11$$

$$x = \frac{\begin{vmatrix} 5 & 7 \\ 11 & 4 \end{vmatrix}}{\begin{vmatrix} 3 & 7 \\ 2 & 4 \end{vmatrix}} = \frac{20-77}{12-14} = \frac{-57}{-2} = \frac{57}{2}$$

$$y = \frac{\begin{vmatrix} 3 & 5 \\ 2 & 11 \end{vmatrix}}{\begin{vmatrix} 3 & 7 \\ 2 & 4 \end{vmatrix}} = \frac{33-10}{-2} = \frac{23}{-2} = -\frac{23}{2}$$

10 points

3. *Let* $\mathbf{P}_2$ be the space of polynomials of degree two or less. The sets $A = \{1 - 2t + t^2, 3 - 5t + 4t^2, 2t + 3t^2\}$ and $B = \{1, t, t^2\}$ from bases for $\mathbf{P}_2$.

(a) Calculate the change-of-coordinate matrix $[P]_{B \leftarrow A}$.

$$\underset{B \leftarrow A}{[P]} = \left([1-2t+t^2]_B \quad [3-5t+4t^2]_B \quad [2t+3t^2]_B \right) = \begin{pmatrix} 1 & 3 & 0 \\ -2 & -5 & 2 \\ 1 & 4 & 3 \end{pmatrix}$$

10 points

(b) Evaluate $[-1 + 2t + t^2]_A$. $= \begin{pmatrix} 11 \\ -4 \\ 2 \end{pmatrix}$

$$-1+2t+t^2 = \alpha(1-2t+t^2) + \beta(3-5t+4t^2) + \gamma(2t+3t^2)$$
$$= (\alpha+3\beta) + (-2\alpha-5\beta+2\gamma)t + (\alpha+4\beta+3\gamma)t^2$$

$\alpha + 3\beta = -1$
$-2\alpha - 5\beta + 2\gamma = 2$
$\alpha + 4\beta + 3\gamma = 1$

$$\alpha = \frac{\begin{vmatrix} -1 & 3 & 0 \\ 2 & -5 & 2 \\ 1 & 4 & 3 \end{vmatrix}}{\begin{vmatrix} 1 & 3 & 0 \\ -2 & -5 & 2 \\ 0 & 4 & 3 \end{vmatrix}} = \frac{11}{1} = 11$$

$$\beta = \frac{-1-\alpha}{3} = -4$$

$$\gamma = \frac{1-\alpha-4\beta}{3} = \frac{1-11-4(-4)}{3} = \frac{6}{3} = 2$$

10 points

Page 2 Total (30)

$$[-1+2t+t^2]_A = \begin{pmatrix} 11 \\ -4 \\ 2 \end{pmatrix}$$

4. Evaluate

(a) $\det\begin{pmatrix} 1 & 8 & -2 & 3 \\ 0 & 2 & -1 & 3 \\ 0 & 0 & -1 & 1/2 \\ 0 & 0 & 0 & 5 \end{pmatrix} = 1\cdot 2\cdot -1\cdot 5 = -10$, prod of diagonals since upper Δ.

(b) $\det\begin{pmatrix} -6 & 0 & 12 & 2 \\ 7 & 0 & 7 & 1 \\ 2 & 0 & 11 & -4 \\ 6 & 0 & -3 & 8 \end{pmatrix} = 0$, Since a column consists entirely of 0's.

(c) $\det(A)$, given $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{pmatrix} A = \begin{pmatrix} 3 & 0 & 3 \\ 0 & 0 & 3 \\ 0 & 3 & 0 \end{pmatrix}$

$$\begin{vmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{vmatrix} |A| = \begin{vmatrix} 3 & 0 & 3 \\ 0 & 0 & 3 \\ 0 & 3 & 0 \end{vmatrix}$$

$$|A| = \frac{\begin{vmatrix} 3 & 0 & 3 \\ 0 & 0 & 3 \\ 0 & 3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{vmatrix}} = \frac{-3\begin{vmatrix} 3 & 3 \\ 0 & 3 \end{vmatrix}}{1\begin{vmatrix} 2 & 3 \\ 5 & 3 \end{vmatrix} + 8\begin{vmatrix} 1 & 2 \\ 2 & 5 \end{vmatrix}} = \frac{-27}{(6-15)+8(5-4)} = \frac{-27}{-9+8} = 27$$

12 pts

5. Is $H = \{A \in M_{2\times 2} |\ tr(A) = a_{11} + a_{22} = 0\}$ a subspace of the vector space $M_{2\times 2}$, the 2×2 matrices?

i) $A = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$, $tr A = 0 + 0 = 0$. So $\underline{0}$ is in H.

ii) $A, B \in H$

$$A+B = \begin{pmatrix} a_{11}+b_{11} & a_{12}+b_{12} \\ a_{21}+b_{21} & a_{22}+b_{22} \end{pmatrix}$$

$$\begin{aligned} tr(A+B) &= (a_{11}+b_{11}) + (a_{22}+b_{22}) \\ &= (a_{11}+a_{22}) + (b_{11}+b_{22}) \\ &= tr A + tr B = 0 + 0 = 0 \end{aligned}$$

So $A+B \in H$.

10 points

iii) $A \in H$

Page 3 Total (22)

$$\alpha A = \begin{pmatrix} \alpha a_{11} & \alpha a_{12} \\ \alpha a_{21} & \alpha a_{22} \end{pmatrix}$$

$$\begin{aligned} tr(\alpha A) &= \alpha a_{11} + \alpha a_{22} \\ &= \alpha(a_{11} + a_{22}) \\ &= \alpha\, tr A = \alpha \cdot 0 = 0 \end{aligned}$$

So $\alpha A \in H$.

6. Consider the vector space $V = \{p(x) \in \mathbf{P}_3 | p(x) = ax^3 + bx^2 + cx + (a+b+c)\}$.

(a) For what value of n is V isomorphic to $\mathbf{R}^n$.

Find a basis

$a=1 \quad x^3+1$

$b=1 \quad x^2+1$

$c=1 \quad x+1$

$\beta = \{x^3+1, x^2+1, x+1\}$

$\underline{n=3}$

5 points

(b) Find an isomporphism from V to $\mathbf{R}^n$.

$T(x^3+1) = e_1 = \begin{pmatrix}1\\0\\0\end{pmatrix}$

$T(x^2+1) = e_2 = \begin{pmatrix}0\\1\\0\end{pmatrix}$

$T(x+1) = e_3 = \begin{pmatrix}0\\0\\1\end{pmatrix}$

- a linear transformation is completely determined by its action on a basis.
- An isomorphism between 2 vectorspaces w/ the same size is takes bases to bases.

Hence $T(ax^3+bx^2+cx+(a+b+c)) = \begin{pmatrix}a\\b\\c\end{pmatrix}$.

8 points

7. Determine if $S = \left\{ \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 3 & 2 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 4 & 0 \\ -1 & 0 \end{pmatrix}, \begin{pmatrix} 11 & 0 \\ 0 & 3 \end{pmatrix} \right\}$ forms a basis for $M_{2\times 2}$.

β = standard basis = $\left\{ \begin{pmatrix}1&0\\0&0\end{pmatrix}, \begin{pmatrix}0&1\\0&0\end{pmatrix}, \begin{pmatrix}0&0\\1&0\end{pmatrix}, \begin{pmatrix}0&0\\0&1\end{pmatrix} \right\}$

writing S as coord vectors in $\mathbb{R}^4$:

$[S]_\beta = \left\{ \begin{pmatrix}2\\0\\0\\0\end{pmatrix}, \begin{pmatrix}3\\2\\0\\0\end{pmatrix}, \begin{pmatrix}4\\0\\-1\\0\end{pmatrix}, \begin{pmatrix}11\\0\\0\\3\end{pmatrix} \right\}$

$\begin{vmatrix} 2 & 3 & 4 & 11 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 3 \end{vmatrix} = 4(-3) = -12 \neq 0$. So S is lin indep.

10 pts

Page 4 Total (23)