

Show ALL your work. Please circle your final answers.

1. Find all the first partial derivatives of the function $f(x, y, z) = e^{x^2y} + \sin(yz)$.

10 Points

2. Find the domain and range and sketch two level curves of $z = \sqrt{9 - x^2 - 3y^2}$.

15 Points

25 Points

3. Given $f(x, y) = \frac{4x^2 + 7y^2}{7x^2 - 2y^2}$, discuss $\lim_{(x, y) \rightarrow (0, 0)} f(x, y)$ and discuss the continuity of $f(x, y)$.

10 Points

4. Classify the critical points of $3x^2y + y^3 - 3x^2 - 3y^2 + 2$.

15 Points

25 Points

5. Given $v = x + \sin(y - z) + w^2$, where $x = s^2$, $y = \ln(s + t)$, $z = s \cos(t)$, and $w = s \sin(t)$, use the chain rule to find $\frac{\partial v}{\partial s}$ and $\frac{\partial v}{\partial t}$ in terms of s and t .

10 Points

6a. The temperature T at any point in a steel ball, centered at the origin, is given by $T(x, y, z) = 360(x^2 + y^2 + z^2)^{-1/2}$. Find the rate of change of T at $(1, 2, 2)$ in the direction toward the point $(2, 1, 3)$.

10 Points

6b. Sketch a level surface of the temperature function.

3 Points

6c. Find the rate of change of T if one travels along a curve lying on this level surface.

2 Points

25 Points

7. Find the points on the ellipsoid $x^2 + 2y^2 + 3z^2 = 1$ at which the tangent plane is parallel to the plane $3x - y + 3z = 1$.

15 Points

- 8a. The two legs of a right triangle are measured as 5 m and 12m, respectively, with a possible error in measurement of at most 0.2 cm in each. Use differentials to estimate the maximum error in the calculated value of the area of the triangle.

7Points

- 8b. Use differentials to estimate the maximum error in the calculated length of the hypotenuse.

8 Points

30 Points