1. Write \( \mathbf{i} + \mathbf{j} + \mathbf{k} \) as the sum of a vector parallel to and orthogonal to \( 4\mathbf{i} + 2\mathbf{j} \).

\[
\mathbf{a} = \mathbf{b} + \mathbf{c} = \left( \begin{array}{c}
2 \\
1 \\
1
\end{array} \right)
\]

\[
\mathbf{b} = \left( \begin{array}{c}
2 \\
1 \\
1
\end{array} \right), \quad \mathbf{c} = \left( \begin{array}{c}
1 \\
1 \\
1
\end{array} \right)
\]

\[
\mathbf{a} \cdot \mathbf{b} = 6, \quad \mathbf{a} \cdot \mathbf{c} = 4, \quad \mathbf{b} \cdot \mathbf{c} = 2
\]

2. Given the vectors \( \mathbf{a}, \mathbf{b}, \) and \( \mathbf{c} \), show, using vector operations, how you would

(a) decide if \( \mathbf{a} \) and \( \mathbf{b} \) are orthogonal

\[
\mathbf{a} \cdot \mathbf{b} = 0 \Rightarrow \mathbf{a} \text{ and } \mathbf{b} \text{ are orthogonal}
\]

(b) decide if \( \mathbf{a} \) and \( \mathbf{c} \) are parallel

\[
\mathbf{a} \times \mathbf{c} = 0 \Rightarrow \mathbf{a} \text{ and } \mathbf{c} \text{ are parallel}
\]

(c) calculate the area of the triangle determined by \( \mathbf{a} \) and \( \mathbf{b} \)

\[
\text{Area} = \frac{1}{2} || \mathbf{a} \times \mathbf{b} ||
\]

(d) calculate the volume of the parallelepiped determined by \( \mathbf{a}, \mathbf{b}, \) and \( \mathbf{c} \)

\[
\text{Volume} = | \mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) |
\]
3. Identify the given surface and convert its equation to cylindrical coordinates:

\[(x^2 - 2x + 1) + y^2 + z^2 = 1\]

\[y^2 + z^2 = 2\]

4. Convert the following equation in spherical coordinates to one in rectangular coordinates and sketch the surface:

\[\rho^4 \sin^2 \phi \cos^2 \theta + \cos^2 \phi = 4\]

Find the distance from the point P(1, 1, 1) to the line through the points Q(0, 6, 0) and R(-1, 4, 3).
6. Find an equation of the plane passing through the point \((1, 6, -4)\) and containing the line \(x + 2y - 3z = 4\). Let \(n = (1, 2, 3)\) be the normal vector to the plane.

7. A particle starts at the point \((0, 1, 0)\) with initial velocity \(\vec{v} = 2\hat{i} + \hat{j} + 3\hat{k}\). Its acceleration is \(\vec{a} = 4\hat{i} + 2\hat{j} - \hat{k}\). Find the particle’s location at time \(t = 2\).

8. As shown in the figure, a ball rolls off a table with a speed of 2 m/s. The table is 3/2 m high. Determine the point \(P\) where the ball hits the floor. For six bonus points, find the angle \(\theta\)!
The position of a projectile is given by \( \mathbf{r}(t) = \mathbf{a} + \ln(\sin(t)) \mathbf{b} + t \mathbf{k} \). Find the following:

a) The projectile's velocity, \( \mathbf{v} \)
\[
\mathbf{v}(t) = \frac{d}{dt} \mathbf{r}(t) = \mathbf{0} \mathbf{i} + \frac{\cos(t)}{\sin(t)} \mathbf{j} + \mathbf{k}
\]

b) The speed of the projectile
\[
|\mathbf{v}(t)| = \sqrt{\left( \frac{\cos(t)}{\sin(t)} \right)^2 + 1} = \sec(t)
\]

\[2 \text{ Points}\]

(c) The unit tangent vector, \( \mathbf{T} \), to the curve
\[
\mathbf{T}(t) = \frac{\mathbf{v}(t)}{|\mathbf{v}(t)|} = \frac{\mathbf{0} \mathbf{i} + \mathbf{j} + \mathbf{k}}{\sec(t)}
\]

\[2 \text{ Points}\]

The unit normal vector, \( \mathbf{N} \), to the curve
\[
\mathbf{N}(t) = \frac{\mathbf{T}'(t)}{|\mathbf{T}'(t)|} = \frac{\mathbf{0} \mathbf{i} - \sin(t) \mathbf{j} + \cos(t) \mathbf{k}}{\sqrt{1 + \sin^2(t)}}
\]

\[4 \text{ Points}\]

(e) The curvature, \( \kappa \), of the curve
\[
\kappa = \left| \frac{d\mathbf{T}}{dt} \right| = \left| \frac{\mathbf{T}'(t)}{|\mathbf{T}'(t)|} \right| = \frac{1}{\sec(t)} = \csc(t)
\]

\[4 \text{ Points}\]

5) The projectile's acceleration written in terms of \( \mathbf{T} \) and \( \mathbf{N} \). You do not have to write out \( \mathbf{T} \) and \( \mathbf{N} \) in your answer.
\[
\ddot{\mathbf{a}} = a_\perp \mathbf{T} + a_\parallel \mathbf{N}
\]
\[
s' = \csc(t)
\]
\[
s'' = -\csc(t) \cot(t)
\]
\[
\ddot{\mathbf{a}} = s' \mathbf{T} + s'' \mathbf{N}
\]
\[
\ddot{\mathbf{a}} = \csc(t) \cot(t) \mathbf{T} + \csc(t) \mathbf{N}
\]
\[
\ddot{\mathbf{a}} = \csc(t) \cot(t) \mathbf{T} + \csc(t) \mathbf{N}
\]

\[6 \text{ Points}\]