CALCULUS III - 3450:223
TEST # 1 NAME_____

100 Points

1. Write $\mathbf{i} + \mathbf{j} + \mathbf{k}$ as the sum of a vector parallel to and orthogonal to $4\mathbf{i} + 2\mathbf{j}$.

10 Points

2. Given the vectors **a**, **b**, and **c**, show, using vector operations, how you would

a) decide if **a** and **b** are orthogonal

2 Points

b) decide if **a** and **c** are parallel

2 Points

c) calculate the area of the triangle determined by **a** and **b**

3 Points

d) calculate the volume of the parallelepiped determined by **a**, **b**, and **c**.

3 Points

20 Points

3. Identify the given surface and convert its equation to cylindrical coordinates:
$x^2 + y^2 + z^2 = 2x$.

8 Points

4. Convert the following equation in spherical coordinates to one in rectangular coordinates and sketch the surface: $\rho^2 [\sin^2\phi \cos^2\theta + \cos^2\phi] = 4$.

8 Points

5. Find the distance from the point P(1, 1, 1) to the line through the points Q(0, 6, 8) and R(-1, 4, 7).

12 Points

28 Points

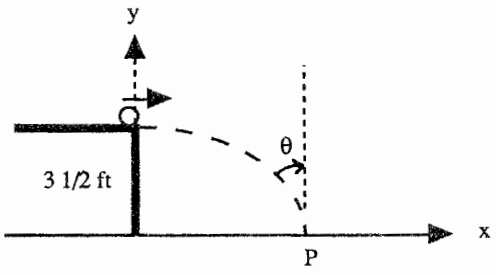
6. Find an equation of the plane passing through the point (1, 6, -4) and containing the line $x = 1 + 2t$, $y = 2 - 3t$, $z = 3 - t$.

12 Points

7. A particle starts at the point (0, 1, 0) with initial velocity $2\mathbf{j}$. Its acceleration is $\mathbf{a}(t) = t\,\mathbf{i} + e^t\,\mathbf{j} + 2\,\mathbf{k}$. Find the particle's location at time $t = 2$.

10 Points

8. As shown in the figure, a ball rolls off of a table with a speed of 2 ft/s. The table is 3 1/2 ft high. Determine the point P where the ball hits the floor. For **six bonus points**, find the angle θ!



10 Points
6 Bonus

38 Points

9. The position of a projectile is given by $\mathbf{r}(t) = 2\mathbf{i} + \ln[\sin(t)]\mathbf{j} + t\,\mathbf{k}$, $\sin(t) > 0$. Find the following :

a) The projectile's velocity, $\mathbf{v}$

2 Points

b) The speed of the projectile

2 Points

c) The unit tangent vector, $\mathbf{T}$, to the curve

2 Points

d) The unit normal vector, $\mathbf{N}$, to the curve

4 Points

e) The curvature, κ, of the curve

4 Points

f) The projectile's acceleration written in terms of $\mathbf{T}$ and $\mathbf{N}$. You do not have to write out $\mathbf{T}$ and $\mathbf{N}$ in your answer.

6 Points

20 Points