Test Total

Name _____

3450:223 Calculus III Final Exam - 150 pts. Spring '02

Clemons

1. Use a transformation to evaluate $\int\int_R \frac{e^{y+3x}}{y-2x} dA$, where R is the rectangle enclosed by the lines $y = 2x + 1$, $y = 2x + 5$, $y = 1 - 3x$ and $y = -1 - 3x$.

12 pts

2. Find the surface area of the portion of the surface $z = 4y + 3x^2$ that lies between $y = 2x$, $y = 0$ and $x = 2$.

11 pts

3. Evaluate the double integral $\int_1^e \int_0^{\ln x} y \, dydx$, by interchanging the order of integration.

11 pts

4. Let D be the solid enclosed by $z = y + 3$, $x^2 + 9z^2 = 9$ and $y = 0$.

(a) Sketch the solid region D.

4 pts

(b) Set-up [**do not solve**] the triple integral in Cartesian coordinates necessary to find the volume of the region D.

7 pts

5. Using spherical coordinates, find the volume of the solid region E bounded by the sphere $x^2 + y^2 + z^2 = 4$ and the planes $z = 0$ and $z = 1$.

11 pts

6. Using cylindrical coordinates, evaluate $\int\int\int_E \frac{y}{x}\, dV$, where E is the region region bounded between $z = x^2 + y^2$ and $z = 0$ and inside $(x-2)^2 + y^2 = 4$.

11 pts

7. Find the volume of the tetrahedron bounded by the coordinate planes and the plane with points $(3, 2, 0)$, $(2, 0, 2)$ and $(0, 0, 3)$.

15 pts

8. Set-up and evaluate using Green's Theorem a line integral for the Total Work done in moving a point through the Force field $\vec{F} = (x^2 y)\vec{i} + x\vec{j}$ around the triangular region bounded by the x-axis, $x = 1$ and the line $y = 2x$, in a counter-clockwise fashion.

10 pts

9. Use a suitable *potential function* to evaluate the line integral
$\int_C (3x + y + 1)\,dx + (x + 4y + 2)\,dy$ where C is any curve connecting $(-1, 2)$ to $(0, 1)$.

10 pts

10. Given that $(0, 0)$, $(-1, 1/2)$ and $(-2, 1)$ are critical points of $f(x, y) = x^3 - 2y^2 - 2y^4 + 3x^2y$, use the second derivative test to classify them.

10 pts

11. Given $f(x, y)$ is a differentiable function with $x = r\cos\theta$ and $y = r\sin\theta$, show that $f_\theta = -f_x r\sin\theta + f_y r\cos\theta$.

9 pts

12. For the space curve $\vec{r}(t) = < e^t, e^{(-t)}, \sqrt{2}\,t >$:
Calculate the arclength for $0 \leq t \leq 1$.

12 pts

13. Given $f(x,y) = e^{-x}\sec y$:

(a) Find the tangent plane at the point $P(0, \pi/4)$.

6 pts

(b) Use the linear approximation to approximate $f(.01, \pi/4 - .01)$.

5 pts

(c) Calculate the directional derivative of $f(x,y)$ at $P(0, \pi/4)$ toward the direction of the origin.

6 pts