① a) $\int \cos(x+1)\,dx = \int \cos u\,du$

$\boxed{\begin{array}{l} u = x+1 \\ du = dx \end{array}}$

$= \sin u + C$

$= \sin(x+1) + C$

b) $\int t \sin t^2\,dt = \frac{1}{2}\int \sin u\,du$

$\boxed{\begin{array}{l} u = t^2 \\ du = 2t\,dt \\ dt = \frac{du}{2t} \end{array}}$

$= -\frac{1}{2}\cos u + C$

$= -\frac{1}{2}\cos t^2 + C$

c) $\int x^9 \sin x^{10}\,dx = \frac{1}{10}\int \sin u\,du$

$\boxed{\begin{array}{l} u = x^{10} \\ du = 10x^9\,dx \\ dx = du/10x^9 \end{array}}$

$= -\frac{1}{10}\cos u + C$

$= -\frac{1}{10}\cos x^{10} + C$

d) $\int \frac{\sin x}{\cos^2 x}\,dx = -\int \frac{du}{u^2}$

$\boxed{\begin{array}{l} u = \cos x \\ du = -\sin x\,dx \\ dx = du/{-\sin x} \end{array}}$

$= \frac{1}{u} + C = \frac{1}{\cos x} + C$

e) $\int y^3\sqrt{y^2+1}\,dy = \frac{1}{2}\int (u-1)\,u^{\frac{1}{2}}\,du = \frac{1}{2}\int \left(u^{\frac{3}{2}} - u^{\frac{1}{2}}\right)du$

$\boxed{\begin{array}{l} u = y^2+1 \\ du = 2y\,dy \\ dy = \frac{du}{2y} \end{array}}$ $\boxed{y^2 = u-1}$

$= \frac{1}{2}\left[\frac{2}{5}u^{5/2} - \frac{2}{3}u^{\frac{3}{2}}\right] + C$

$= \frac{1}{5}(y^2+1)^{5/2} - \frac{1}{3}(y^2+1)^{3/2} + C$

(2)

a) $\int \frac{x\,dx}{\cos^2(x^2+5)} = \frac{1}{2}\int \frac{du}{\cos^2 u} = \frac{1}{2}\int \sec^2 u\,du$

$\boxed{\begin{array}{l} u = x^2+5 \\ du = 2x\,dx \\ dx = du/2x \end{array}}$

$= \frac{1}{2}\tan u + C$

$= \frac{1}{2}\tan(x^2+5) + C$

b) $\int (2x^2+2)\sqrt{x^3+3x}\,dx = \int \frac{2(x^2+1)}{3(x^2+1)}\sqrt{u}\,du$

$\boxed{\begin{array}{l} u = x^3+3x \\ du = (3x^2+3)dx \end{array}}$

$= \frac{2}{3}\int u^{\frac{1}{2}}du = \left(\frac{2}{3}\right)^2 u^{\frac{3}{2}} + C$

$= \frac{4}{9}(x^3+3x)^{\frac{3}{2}} + C$

c) $\int \frac{dx}{(3x-1)^{1/3}} = \frac{1}{3}\int \frac{du}{u^{1/3}} = \frac{1}{3}\int u^{-1/3}du = \frac{1}{3}\cdot\frac{3}{2}u^{\frac{2}{3}} + C$

$\boxed{\begin{array}{l} u = 3x-1 \\ du = 3dx \end{array}}$

$= \frac{1}{2}(3x-1)^{2/3} + C$

d) $\int \sin 3x \cos^2 3x\,dx = -\frac{1}{3}\int u^2 du$

$\boxed{\begin{array}{l} u = \cos 3x \\ du = (-\sin 3x)(3) \end{array}}$

$= -\frac{1}{3}\cdot\frac{1}{3}u^3 + C = -\frac{1}{9}(\cos 3x)^3 + C$

e) $\int_1^2 (z+1)\sqrt{z-1}\,dz = \int_0^1 (u+2)u^{\frac{1}{2}}du = \int_0^1 u^{\frac{3}{2}} + 2u^{\frac{1}{2}}du$

$\boxed{\begin{array}{l} u = z-1 \Rightarrow z = u+1 \\ du = dz \end{array}}$

$= \frac{2}{5}u^{5/2} + 2\cdot\frac{2}{3}u^{3/2}\Big|_0^1 = \frac{2}{5} + \frac{4}{3} = \frac{6+20}{15} = \frac{26}{15}$

f) $\int \frac{\sqrt{\sqrt{x}+1}}{\sqrt{x}}\,dx = \int 2\sqrt{u}\,du = 2\int u^{\frac{1}{2}}\,du$

$\boxed{\begin{aligned} u &= \sqrt{x}+1 \\ du &= \tfrac{1}{2}x^{-1/2}dx \\ dx &= 2x^{\frac{1}{2}}du \end{aligned}}$

$= 2\cdot\frac{2}{3}u^{3/2} = \frac{4}{3}(\sqrt{x}+1)^{3/2} + C$

g) $\int_0^4 \sqrt{x}\sqrt{x\sqrt{x}+1}\,dx = \frac{2}{3}\int_1^9 u^{\frac{1}{2}}\,du = \frac{2}{3}\cdot\frac{2}{3}u^{3/2}\Big|_1^9$

$\boxed{\begin{aligned} u &= x\sqrt{x}+1 = x^{\frac{3}{2}}+1 \\ du &= \tfrac{3}{2}x^{\frac{1}{2}}dx \\ dx &= \frac{2\,du}{3\sqrt{x}} \end{aligned}}$

$= \frac{4}{9}(27-1) = \frac{4\cdot 26}{9} = \frac{104}{9}$

h) $\int_{\frac{\pi^2}{16}}^{\frac{\pi^2}{4}} \frac{\cos\sqrt{x}\,dx}{\sqrt{x}\,\sin^3\sqrt{x}} = \frac{1}{2}\int_{\frac{1}{\sqrt{2}}}^{1} u^{-3}\,du = \frac{1}{2}\left(\frac{-u^{-2}}{2}\right)\Big|_{\frac{1}{\sqrt{2}}}^{1} = \frac{1}{2}\left(\frac{-1}{2}+\frac{2}{2}\right) = \frac{1}{4}$

$\boxed{\begin{aligned} u &= \sin\sqrt{x} \\ du &= \frac{1}{2}\cdot\frac{\cos\sqrt{x}}{\sqrt{x}}\,dx \end{aligned}}$

i) $\int \frac{dx}{x^2+2x+1} = \int \frac{dx}{(x+1)^2} = \int u^{-2}\,du = -u^{-1} = -(x+1)^{-1} + C$

$\boxed{\begin{aligned} u &= x+1 \\ du &= dx \end{aligned}}$

$= \frac{-1}{(x+1)} + C$

j) $\int x^2(x^4-7)^2\,dx = \int x^2(x^8-14x^4+49)\,dx = \int(x^{10}-14x^6+49x^2)\,dx$

$= \frac{x^{11}}{11} - \frac{14x^7}{7} + \frac{49x^3}{3} + C$

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k) $\int \frac{x^4-7x^2+3}{x^2}\,dx = \int\left(x^2-7+\frac{3}{x^2}\right)dx$

$= \frac{x^3}{3} - 7x - \frac{3}{x} + C$

3) $\int(6x^2-1)\,dx = 2x^3-x+C$

$f(2) = 16-2+C = 10 \Rightarrow C=-4$

$\Rightarrow f(x) = 2x^3-x-4$

4) $\int_2^5 (x^2-2x+3)\,dx = \lim_{n\to\infty} \frac{3}{n}\sum_{i=1}^{n} f\left(2+\frac{3i}{n}\right) = \lim_{n\to\infty} \frac{3}{n}\sum_{i=1}^{n}\left(\left(2+\frac{3i}{n}\right)^2 - 2\left(2+\frac{3i}{n}\right)+3\right)$

$= \lim_{n\to\infty}\frac{3}{n}\sum_{i=1}^{n}\left(\frac{6i}{n}+\frac{9i^2}{n^2}+3\right) = \lim_{n\to\infty}\frac{3}{n}\left[\frac{6n(n+1)}{2n}+\frac{9}{n^2}\cdot\frac{n(n+1)(2n+1)}{6}+3n\right]$

$= \lim_{n\to\infty}\left[\frac{9(n(n+1))}{n^2} + \frac{27}{6}\left[\frac{2n^3+3n^2+n}{n^3}\right]+9\right]$

$= \lim_{n\to\infty}\left[9\left(1+\frac{1}{n^2}\right)+\frac{27}{6}\left(2+\frac{3}{n}+\frac{1}{n^2}\right)+9\right] = 9+9+9 = 27$

5) a) $\frac{d}{dx}\int_4^x \cos t \sin t\,dt = \cos x \sin x$

b) Let $u = t^2 \Rightarrow \frac{du}{dt} = 2t$

$\frac{d}{du}\int_1^u \frac{x^2+1}{\sqrt{x}}\,dx\ \frac{du}{dt} = \frac{u^2+1}{\sqrt{u}}\cdot\frac{du}{dt} = \frac{t^4+1}{\sqrt{t^2}}\cdot 2t$

c) Let $u=\sin z \Rightarrow \frac{du}{dz} = \cos z$, $v = z^2 \Rightarrow \frac{dv}{dz} = 2z$

$\frac{d}{dz}\int_{z^2}^{\sin z} t\cos t^3\,dt = \frac{d}{du}\int_0^u t\cos t^3\,dt\cdot\frac{du}{dz} - \frac{d}{du}\int_0^v t\cos t^3\,dt\cdot\frac{dv}{dz} = \sin z\cos(\sin^3 z)\cos z$
$-2z^3\cos(z^6)$

⑥ a) $\int_1^2 \left(x+\frac{1}{x}\right)^2 dx = \int_1^2 \left(x^2+2+\frac{1}{x^2}\right) dx$

$= \frac{x^3}{3} + 2x - \frac{1}{x} \Big|_1^2 = \left(\frac{8}{3}+4-\frac{1}{2}\right) - \left(\frac{1}{3}+2-1\right) = \frac{29}{6}$

b) $\int_{3\pi/8}^{\pi/2} \sec 2x \tan 2x \, dx = \frac{\sec 2x}{2} \Big|_{3\pi/8}^{\pi/2} = \frac{1}{2}\left(-1+\sqrt{2}\right)$

$\boxed{\begin{array}{c} u = 2x \\ du = 2\,dx \end{array}}$ $= \frac{\sqrt{2}}{2} - \frac{1}{2}$

c) $\int_0^1 (y^9 - 2y^5 + 3y)\,dy = \frac{y^{10}}{10} - \frac{2y^6}{6} + \frac{3y^2}{2} \Big|_0^1$

$= \frac{1}{10} - \frac{1}{3} + \frac{3}{2} = \frac{19}{15}$

d) $f(x) = \begin{cases} x^2 - \frac14 & \text{if } x \le -\frac12 \text{ or } x \ge \frac12 \\ \frac14 - x^2 & \text{if } -\frac12 < x < \frac12 \end{cases}$

$\int_{-1}^{2} \left|x^2 - \frac14\right| dx = \int_{-1}^{-1/2} \left(x^2 - \frac14\right) dx + \int_{-1/2}^{1/2} \left(\frac14 - x^2\right) dx + \int_{1/2}^{2} \left(x^2 - \frac14\right) dx$

$= \frac{x^3}{3} - \frac{x}{4} \Big|_{-1}^{-1/2} + \frac{x}{4} - \frac{x^3}{3} \Big|_{-1/2}^{1/2} + \frac{x^3}{3} - \frac{x}{4} \Big|_{1/2}^{2}$

$= -\cancel{\frac{1}{24}} + \cancel{\frac{1}{8}} - \left(\frac{-1}{3} + \frac{1}{4}\right) + \cancel{\frac{1}{8}} - \cancel{\frac{1}{24}} - \left(\cancel{\frac{-1}{8}} + \cancel{\frac{1}{24}}\right) + \frac{8}{3} - \frac{1}{2} - \cancel{\frac{1}{24}} + \cancel{\frac{1}{8}}$

$= \frac{1}{2} - \frac{1}{6} + \frac{1}{3} - \frac{1}{4} + \frac{8}{3} - \frac{1}{2} = -\frac{1}{6} + \frac{9}{3} - \frac{1}{4} = \frac{31}{12}$

e) $\int x^2 (1-x^3)^8 dx = -\frac{1}{3} \int u^8 du = -\frac{1}{3} \frac{u^9}{9}$

$\boxed{\begin{array}{c} u = 1 - x^3 \\ du = -3x^2 dx \end{array}}$ $= -\frac{1}{27} (1-x^3)^9 + C$

f) $\int \frac{x\,dx}{\sqrt{x^2+1}} = \frac{1}{2}\int u^{-\frac{1}{2}}\,du = u^{\frac{1}{2}} = (x^2+1)^{\frac{1}{2}} + C$

$\boxed{\begin{array}{l} u = x^2+1 \\ du = 2x\,dx \end{array}}$

g) $\int_0^8 \frac{\cos(x+1)^{\frac{1}{2}}}{(x+1)^{\frac{1}{2}}}\,dx = 2\int \cos u\,du = 2\sin u$

$\boxed{\begin{array}{l} u = (x+1)^{\frac{1}{2}} \\ du = \frac{1}{2}(x+1)^{-\frac{1}{2}}\,dx \end{array}}$

$= 2\sin(x+1)^{\frac{1}{2}}\Big|_0^8$

$= 2\sin(3) - 2\sin(1)$

h) $\int_1^4 \frac{1}{x^2}\left(1+\frac{1}{x}\right)^{\frac{1}{3}}dx = -\int_2^{\frac{5}{4}} u^{\frac{1}{3}}\,du = \frac{-3u^{\frac{4}{3}}}{4}\Big|_2^{\frac{5}{4}}$

$\boxed{\begin{array}{l} u = 1+\frac{1}{x} \\ du = \frac{-dx}{x^2} \end{array}}$

$= -\frac{3}{4}\left(\frac{5}{4}\right)^{\frac{4}{3}} + \frac{3}{4}(2)^{\frac{4}{3}} \approx 0.88$

• i) $\int_0^4 \frac{x\,dx}{\sqrt{1+2x}} = \frac{1}{4}\int_1^9 (u-1)u^{-\frac{1}{2}}\,du = \frac{1}{4}\int_1^9 u^{\frac{1}{2}} - u^{-\frac{1}{2}}\,du$

$\boxed{\begin{array}{l} u = (1+2x) \Rightarrow x = \frac{u-1}{2} \\ du = 2\,dx \end{array}}$

$= \frac{1}{4}\cdot\frac{2}{3}u^{\frac{3}{2}} - \frac{1}{4}\cdot 2u^{\frac{1}{2}}\Big|_1^9$

$= \frac{1}{6}(27) - \frac{1}{2}(3) - \frac{1}{6} + \frac{1}{2} = \frac{26}{6} - \frac{6}{6} = \frac{20}{6}$

j) $\int \frac{x^2\,dx}{(1-x)^{\frac{1}{2}}} = -\int \frac{(u-1)^2}{u^{\frac{1}{2}}} = -\int \frac{u^2-2u+1}{u^{\frac{1}{2}}}\,du = -\int u^{\frac{3}{2}} - 2u^{\frac{1}{2}} + u^{-\frac{1}{2}}\,du$

$\boxed{\begin{array}{l} u = 1-x \\ du = -dx \end{array}}$

$= \frac{-2u^{\frac{5}{2}}}{5} + \frac{2\cdot 2}{3}u^{\frac{3}{2}} - 2u^{\frac{1}{2}} = \frac{-2}{5}(1-x)^{\frac{5}{2}} + \frac{4}{3}(1-x)^{\frac{3}{2}}$

$-2(1-x)^{\frac{1}{2}} + C$

1 a) $\int_{-2}^{-2} f(x)\,dx - \int_{-2}^{5} f(x)\,dx = 0-(-5) = 5$

b) $\int_{-2}^{10} (2f(x) - 3g(x))\,dx = 2\int_{-2}^{10} f(x)\,dx - 3\int_{-2}^{10} g(x)\,dx$

$= 2(7) - 3(12) = -22$

c) Note:

$\int_{-2}^{10} f(x)\,dx = \int_{5}^{10} f(x)\,dx + \int_{-2}^{5} f(x)\,dx$

$\Rightarrow \int_{5}^{10} f(x)\,dx = \int_{-2}^{10} f(x)\,dx - \int_{-2}^{5} f(x)\,dx$

$= 7-(-5) = 12$

8) $\int_{-3}^{2} (2x^2 - 4x + 5)\,dx$