

## Integration Problems for Fun and Profit!

1. Decide which integral in each pair you can evaluate and do so:

(a)  $\int (x+1) \cos(x+1) dx$  versus  $\int \cos(x+1) dx$

(b)  $\int t \sin t^2 dt$  versus  $\int \sin t^2 dt$

(c)  $\int x^{10} \sin x^9 dx$  versus  $\int x^9 \sin x^{10} dx$

(d)  $\int \frac{\sin x}{\cos^2 x} dx$  versus  $\int \frac{\sin^2 x}{\cos x} dx$

(e)  $\int y^3 \sqrt{y^2+1} dy$  versus  $\int y^2 \sqrt{y^2+1} dy$

2. Evaluate each of these integrals:

(a)  $\int \frac{x}{\cos^2(x^2+5)} dx$

(b)  $\int (2x^2+2)\sqrt{x^3+3x} dx$

(c)  $\int \frac{dx}{(3x-1)^{1/3}}$

(d)  $\int \sin 3x \cos^3 3x dx$

(e)  $\int_1^2 (z+1)\sqrt{z-1} dz$

(f)  $\int \frac{\sqrt{\sqrt{x}+1}}{\sqrt{x}} dx$

(g)  $\int_0^4 \sqrt{x} \sqrt{x\sqrt{x}+1} dx$

(h)  $\int_{\pi^2/16}^{\pi^2/4} \frac{\cos \sqrt{x}}{\sqrt{x} \sin^3 \sqrt{x}} dx$

(i)  $\int \frac{dx}{x^2+2x+1}$

(j)  $\int x^2(x^4-7)^2 dx$

(k)  $\int \frac{x^4-7x^2+3}{x^2} dx$

3. Find the function  $f(x)$  whose graph passes through the point  $(2,10)$  and whose derivative is  $f'(x) = 6x^2 - 1$ .

4. Use Riemann Sums to evaluate  $\int_2^5 x^2 - 2x + 3 dx$

(over)

5. Differentiate the following functions:

(a)  $F(x) = \int_4^x \cos t \sin t \, dt$

(b)  $G(t) = \int_1^{t^2} \left( \frac{x^2+1}{\sqrt{x}} \right) dx$

(c)  $H(z) = \int_{z^2}^{\sin z} t \cos t^3 \, dt$

6. Evaluate the integrals:

(a)  $\int_1^2 \left( x + \frac{1}{x} \right)^2 dx$

(b)  $\int_{3\pi/8}^{\pi/2} \sec 2x \tan 2x \, dx$

(c)  $\int_0^1 (y^9 - 2y^5 + 3y) \, dy$

(d)  $\int_{-1}^2 |x^2 - 1/4| \, dx$

(e)  $\int x^2(1 - x^3)^8 \, dx$

(f)  $\int \frac{x}{\sqrt{x^2+1}} \, dx$

(g)  $\int_0^8 \frac{\cos \sqrt{x+1}}{\sqrt{x+1}} \, dx$

(h)  $\int_1^4 \frac{1}{x^2} \left( 1 + \frac{1}{x} \right)^{1/3} \, dx$

(i)  $\int_0^4 \frac{x}{\sqrt{1+2x}} \, dx$

(j)  $\int \frac{x^2}{\sqrt{1-x}} \, dx$

7. Suppose  $f$  and  $g$  are continuous and  $\int_{-2}^5 f(x) \, dx = -5$ ,  $\int_{-2}^{10} f(x) \, dx = 7$ ,  $\int_{-2}^{10} g(x) \, dx = 12$ . Evaluate:

(a)  $\int_{-2}^{-2} f(x) \, dx - \int_{-2}^5 f(x) \, dx$

(b)  $\int_{-2}^{10} (2f(x) - 3g(x)) \, dx$

(c)  $\int_5^{10} f(x) \, dx$

8. Write the integral defined by  $A = \lim_{n \rightarrow \infty} \left[ 2 \left( -3 + \frac{5i}{n} \right)^2 - 4 \left( -3 + \frac{5i}{n} \right) + 5 \right] \frac{5}{n}$ .