

NAME _____

100 Points

Show all your work.

1. Find the derivative of the following functions. **DO NOT** simplify your answer.

a. (7 pts) $f(x) = (2x^3 + 1)^4 \tan(3x^2 - \sqrt{x})$

b. (7 pts) $f(x) = \sec^2(\sin(2x - 1)^5)$

14 Points

2. (11 pts) Find y'' if $x^3 - y^3 = 5$

11 Points

3. (7 pts) Evaluate $\lim_{x \rightarrow 0} \left[\frac{\sin^2 7x}{\sin^2 5x} \right]$

7 Points

4. (9 pts) Use differentials or the equivalent linearization to approximate $\tan(59^\circ)$.

9 Points

5. (8 pts) Find a formula for the n^{th} derivative of $y = \frac{3}{(1 + 2x)^4}$

8 Points

6. (10 pts) Show that the families of curves given by $y = ax^3$ and $x^2 + 3y^2 = b$ are orthogonal trajectories of each other.

10 Points

7. A particle moves according to a law of motion $s(t) = 2t^3 - 3t^2$, $t \geq 0$ where t is measured in seconds and s in feet.

a. (2 pts) Find the velocity at time t .

b. (2 pts) Find the acceleration at time t .

c. (4 pts) Find the time(s) when the particle is at rest.

d. (3 pts) Find when the particle is moving in the positive direction.

e. (4 pts) Find the acceleration when the velocity is 12 feet per second.

15 Points

8. (6 pts) Find the critical numbers of $f(x) = 2x - 3x^{2/3}$

6 Points

9. (10 pts) Find the absolute maximum and absolute minimum values (label each) of

$$f(x) = 3x^4 - 4x^3 \text{ on } [-1, 2]$$

10 Points

10. (10 pts) Sand is falling off a conveyor and onto a conical pile at the rate of $10\text{ft}^3/\text{min}$. The diameter of the base of the cone is 3 times the altitude. At what rate is the height of the pile changing when it is 15 feet high?

$$\left(V = \frac{1}{3}\pi r^2 h\right)$$

10 Points