Review Sheet

Exam III Review Session  Calculus I  3450:221-2005

1. Evaluate \( \lim_{x \to -\infty} \sqrt{4x^2 + 3x - 8 + 2x} \).

2. Use the given information to find \( f(x) \): \( f''(x) = 12x^2 - 6x + 8 - 3 \sin x \), \( f(0) = 4 \), \( f'(0) = -6 \).

3. A one-mile race track is to be built with two straight sides and semicircles at the ends. What is the maximum amount of area needed to construct the track.

4. Sketch the graph of the function \( f(x) \) that satisfies the following conditions: \( f(0) = 0 \); \( f'(-3) = f'(2) = f'(7) = 0 \); \( \lim_{x \to -\infty} f(x) = 0 \); \( \lim_{x \to 5} f(x) = -\infty \); \( f'(x) < 0 \) on \( (-\infty, -3) \cup (2, 5) \cup (7, \infty) \), \( f'(x) > 0 \) on \( (-3, 2) \cup (5, 7) \); \( f''(x) > 0 \) on \( (-\infty, 0) \cup (10, \infty) \); \( f''(x) < 0 \) on \( (0, 5) \cup (5, 10) \).

5. The Boondoccia Outfitting Company has decided to produce and market a small backpacking tent. The have hired you to determine the dimensions of the tent requiring the least amount of material that satisfies the following conditions: (1) the tent must have two sides, two ends, and a bottom; (2) the volume of the tent must be 100 \( ft^3 \); (3) the tent must be 8 \( ft \) long; (4) cross sections of the tent parallel to each end must be congruent isosceles triangles. Set-up the problem you would optimize to use the least materials.

6. Suppose that \( f(x) = rx^2 + sx + d \) for arbitrary constants \( r \), \( s \), and \( d \). Let \( x \in [a, b] \) where \( a \) and \( b \) are constants. Find the number \( c \) guaranteed by the Mean Value Theorem.

7. Suppose \( a \) and \( b \) are constants with \( a > 0 \) and \( b > 0 \). Find the information asked below for the function \( f(x) = b - \frac{a}{x} - \frac{a}{x^2} \):
   (a) Find the horizontal and vertical asymptotes.
   (b) Find the intervals where \( f(x) \) is increasing and decreasing, and identify the relative extrema.
   (c) Find the intervals where \( f(x) \) is concave up and down, and identify the points of inflection.
8. Evaluate the following:
   (a) \( \int_{-3}^{4} | -2x + 4 | \, dx \)
   (b) \( \int \left( t + \frac{1}{t} \right)^2 \, dt \)
   (c) \( \frac{d}{dx} \int_{3x^2+1}^{0} 2t(t^2 + 1) \, dt \)

9. Consider \( \int_{-1}^{1} 2x + 3 \, dx \).
   (a) Write the Riemann sum for \( n \) rectangles using uniform partition.
   (b) Evaluate your Riemann sum in part (a), then take the limit as \( n \to \infty \).

10. (#27 pg. 248) Given the graph \( f'(x) \) of a continuous function \( f(x) \)

(a) On what intervals is \( f(x) \) increasing or decreasing?
(b) At what values of \( x \) does \( f(x) \) have a local maximum or minimum?
(c) On what intervals is \( f(x) \) concave upward or downward?
(d) State the \( x \)-coordinate(s) of the points of inflection.
(e) Assuming that \( f(0) = 0 \), sketch a graph of \( f(x) \).