

Calc I - 3450:221

FINAL

NAME \_\_\_\_\_

ROW \_\_\_\_\_

200 Points

Show **ALL** your work.

1. Find the average value of the function  $f(x) = x^2 - 4x + 5$  over  $x \in [-1, 2]$ .

10 Points

2. Find the area of the region bounded by  $y = -x$  and  $y = x - x^2$ .

10 Points

20 Points

3. Find  $\frac{dy}{dx}$  if  $y = \sqrt{\frac{5x^2 - 7}{9 - 2x^3}}$ . You do not have to simplify your answer.

7 Points

4. Find  $y''$  if  $y = \cos(3x^2)$ .

8 Points

5. Find the equation of the line normal to  $5x^3 - 2x^2y^2 + 4y^3 - 29 = 0$  at the point  $(1, 2)$ .

10 Points

25 Points

6. Consider the region bounded by the graphs of  $y = -(x + 1)$  and  $x = -(y - 1)^2$  for  $y \geq 1$ .

SET UP THE INTEGRAL(S) needed to find the volume of the solid of revolution formed by

revolving this region about the

a) x-axis (Set up the integral(s) for integration with respect to y.)

10 Points

b) x-axis (Set up the integral(s) for integration with respect to x.)

10 Points

c) line  $x = 2$  (Indicate the method used.)

10 Points

d) line  $y = -2$  (Indicate the method used.)

10 Points
40 Points

7. Evaluate  $\int_0^1 \frac{x^2 - 3x + 2}{\sqrt{x}} dx$ .

4

10 Points

8. Evaluate  $\int_{-2}^6 (3 - 7x^5)^3 dx + \int_6^{-2} (3 - 7x^5)^3 dx$ .

10 Points

9. Evaluate  $\int \frac{2 - 6\sin(3x)}{[x + \cos(3x)]^2} dx$ .

10 Points

30 Points

12. Evaluate  $\lim_{n \rightarrow \infty} \sum_{i=1}^n \left(\frac{i}{n}\right)^9 \frac{1}{n}$ .

10 Points

13. A solid has as its base the region in the plane bounded by  $y = x$ ,  $x = 4$  and  $y = -2x$ . Each cross section perpendicular to the plane and parallel to the  $y$ -axis is a rectangle with height four times the length of the base which lies in the plane. Find the volume of this solid.

10 Points

20 Points

14. Answer all of the questions below for  $f(x) = \frac{x^3 - 1}{x^3 + 1}$ .

a) Find any vertical, horizontal, and slant asymptotes. Show all limits used.

7 Points

b) Find the intervals where  $f(x)$  is increasing/decreasing and identify any extrema.

8 Points

c) Find the intervals where  $f(x)$  is concave up/down and identify any points of inflection.

10 Points

25 Points

15. A water tank is in the shape of an inverted cone with radius 15 feet and depth 45 feet. If water is pouring into the tank at the rate of 10 cubic feet per minute, at what rate is the depth of the water changing when the water is 25 feet deep?

10 Points

16. Find the dimensions of the rectangle of largest area that can be inscribed in a circle of radius  $\sqrt{2}$ .

10 Points

20 Points