1. Use Part I of the Fundamental Theorem of Calculus to find the derivative of \( y = \int_0^1 \frac{1}{\sqrt{t^2 + 5}} \, dt \)

\[
\left[ \int_0^1 \frac{1}{\sqrt{t^2 + 5}} \, dt \right]' = - \left[ \int_0^1 \frac{1}{\sqrt{t^2 + 5}} \, dt \right] = - \frac{1}{\sqrt{(0)^2 + 5}} \cdot \frac{1}{\partial t^{1/2}}
\]

2 points

2. Estimate the area under the curve of \( f(x) = 2x^2 + x \) from \( x = -5 \) to \( x = -2 \) using 3 sub-intervals and the right endpoints.

\[
\Delta x = \frac{-2 - (-5)}{3} = 1
\]

\[
\Delta x \left[ f(-4) + f(-3) + f(-2) \right] = 1 \left[ 2(-4)^2 + (-4) + 2(-3)^2 + (-3) + 2(-2)^2 + 2(-2) \right]
\]

\[
= \left( 6 + 15 + 14 \right) = 35
\]

5 points

3. A particle moves on the x-axis with acceleration \( a(t) = \cos(t) - 30t \) m/s². If the velocity at time \( t = 0 \) seconds is \( v(0) = 3 \) m/s, and the position at time \( t = 0 \) seconds is \( s(0) = \frac{5}{2} \) m, find the position at time \( t \).

\[
Q = \cos(t) - 30t
\]

\[
\dot{v} = -s(t) = -15t + C_1
\]

\[
v(0) = 0 = 0 + C_1 \Rightarrow C_1 = 0
\]

\[
\ddot{s} = -\cos(t) - 5e^{3t} C_1 + C_2
\]

\[
s(0) = \frac{5}{2} = -1 + C_2 \Rightarrow C_2 = 1 + \frac{5}{2}
\]

\[
s = -\cos(t) - 5e^{3t} + 3e^{(1 + \frac{5}{2})t}
\]

9 points
4. Sketch the graph of the function $f(x)$ that has the following properties:

- $f(-2) = 0$; $f(0) = 3$; $f(1) = 2$; $f(2) = 1$;
- $\lim_{x \to -\infty} f(x) = -1$; $\lim_{x \to +\infty} f(x) = 4$; $\lim_{x \to 2} f(x) = +\infty$;
- $f'(x) > 0$ on $(-\infty, 0) \cup (2, 3)$;
- $f''(x) < 0$ on $(0, 2) \cup (3, \infty)$;
- $f'(x) > 0$ on $(-\infty, -2) \cup (1, 3) \cup (3, \infty)$;
- $f'''(x) < 0$ on $(-2, 1)$.
5. Given \( f'(x) = \frac{x^2(3-x)}{1+x} \) and \( f''(x) = \frac{2x(3-x^3)}{(1+x)^2} \), find the following:

(a) The intervals where \( f(x) \) is increasing, and those where it is decreasing.

\[
\begin{array}{c|c|c|c|c|c}
 x & -1 & 0 & 1 & x^2 & 3 \\
 \hline
 f' & + & 0 & - & + & - \\
 \hline
end of table
\]

Critical points: \((-1,0), (0, 1), 1 \)  

Local maxima: \( x = 0 \)  

(b) Find the \( x \)-coordinates of any local extrema, and identify whether they are maxima or minima.

\[ \text{local max} \quad x = 0 \]

3 points

(c) Find the intervals on which \( f(x) \) is concave up, and those where it is concave down.

\[
\begin{array}{c|c|c|c|c|c}
 x & -3 & -1 & 0 & 1 & 3 & \infty \\
 \hline
 f'' & - & 0 & + & - & + & 0 \\
 \hline
end of table
\]

Concave up: \((-\infty, -1), (1, \infty)\)  

Concave down: \((-1, 0), (0, 1)\)

6 points

(d) Find the \( x \)-coordinates of any points of inflection.

Note: \( f(x) \) is not continuous.

\[ \text{inf pts} \quad x = -1, 0, 1 \]

3 points

Page 3 Total (18 pts)
6. Evaluate the following limits:

(a) \[
\lim_{x \to -\infty} \frac{5x^2 - 2x + 11}{\sqrt{36x^2 + 3x}} \cdot \frac{\frac{1}{x}}{\sqrt{\frac{1}{x} - \frac{36x + 3}{x^2}}} = \lim_{x \to -\infty} \frac{5 - \frac{2}{x} + \frac{11}{x^2}}{\sqrt{36 + \frac{3}{x}}} = -5
\]

(b) \[
\lim_{t \to \infty} \left( u - 1 - \sqrt{16t^2 - 8t} \right) \cdot \frac{(4t+1 + \sqrt{16t^2 - 8t})}{(4t+1 + \sqrt{16t^2 - 8t})} = \lim_{t \to \infty} \frac{4t^2 - 5 - (16t^2 - 8t)}{4t+1 + \sqrt{16t^2 - 8t}} = \lim_{t \to \infty} \frac{4t^2 - 5 - 16t^2 + 8t}{4t+1 + \sqrt{16t^2 - 8t}} = \lim_{t \to \infty} \frac{-12t + 1}{4t+1 + \sqrt{16t^2 - 8t}} = 0
\]

(c) \[
\lim_{n \to \infty} \sum_{i=1}^{n} \left[ 3 \left( 1 + \frac{i}{n} \right)^2 + 1 \right] \cdot \frac{1}{n} = \int_{1}^{3} \left( x^2 + 1 \right) \, dx = \left[ x^3 + x \right]_{1}^{3} = (27 + 3) - (1 + 1) = 23
\]
7. Evaluate the following:

(a) \[ \int \frac{x^3 - 1}{\sqrt{x}} \, dx = \int x^3 \frac{1}{\sqrt{x}} - \frac{1}{\sqrt{x}} \, dx \]
\[ = \frac{2}{3} x^{3/2} - 2x^{1/2} \bigg|_1 \]
\[ = \frac{2}{3} (4^{3/2} - 2) - (\frac{2}{3} - 2) \]
\[ = \frac{2}{3} (\frac{4}{3}) = \frac{8}{9} \]

(b) \[ \int \sin(x) \cos(\cos(x)) + x^{1/3} \, dx \]
\[ = 0 \]

(c) \[ \int_{-13}^{13} f(x) \, dx, \text{ if } \int_{-13}^{1} f(x) \, dx = -38 \text{ and } \int_{1}^{13} f(x) \, dx = -96. \]
\[ \int_{-13}^{13} f(x) \, dx = \int_{-13}^{1} f(x) \, dx + \int_{1}^{13} f(x) \, dx = -38 + (-96) = 134 \]

8. Suppose that we wish to construct a rectangular box with the base in the ratio 2:1. The volume of the box is 36 in\(^3\). Find the dimensions that minimize the material to make it.

\[ V = 2x^2h = 36 \]
\[ A = 2x^2 + 2(2xh) \]
\[ A(x) = 2x^2 + 4xh \]
\[ A'(x) = 4x + 4h \frac{dx}{dx} = 4x - 108 = 0 \]
\[ x = \frac{27}{2} \]
\[ h = \frac{108}{27} = 2 \]

\[ x = 2, \, 2x = 6, \, h = 2 \]