In this course we will explore some of the key ideas of the theory of finite and infinite groups in some depth. After reviewing homomorphisms and isomorphisms, we will develop the theory of group actions and use these to prove the Sylow theorems. We will then use the Sylow theorems to prove certain groups are not simple. We will see nice results about symmetric and alternating groups, and prove that $A_n$ is simple for $n \geq 5$.

We will also dissect the general linear group $GL(n, q)$, which is the set of all invertible $n$ by $n$ matrices over a field with $q$ elements. These groups arise throughout mathematics, and the powerful tools of group theory allow us to say interesting things about these groups, their subgroups, and some related factor groups. Moreover, these groups arise in many applications of algebra, including physics and chemistry, and are related to the groups of Lie type. We will prove the Bruhat decomposition of $GL(n, q)$, and prove the simplicity of the related group $PSL(n, q)$.

Time permitting, we will explore other topics in group theory, including but not limited to the theory of Polya enumeration (which connects group theory to combinatorics), transfer theory, or the Schur-Zassenhaus theorem. Other topics may be covered depending on student interest.

The primary prerequisite for this graduate-level course is Math 411 or 511. Math 412 or 512 would be helpful but is not necessary. Also Math 410 or 510 (Advanced Linear Algebra) would be helpful but is not necessary (though some linear algebra will be needed). If you have any questions, please feel free to contact me at cossey@uakron.edu, or, better yet, stop by my office (CAS 234). Please note: if you plan on registering for this class, please contact me.