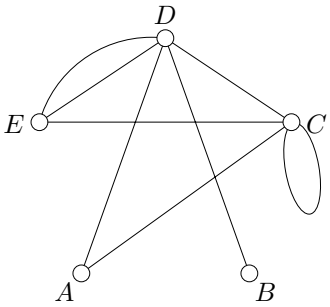
3450:135 Excursions in Mathematics Fall, 2004

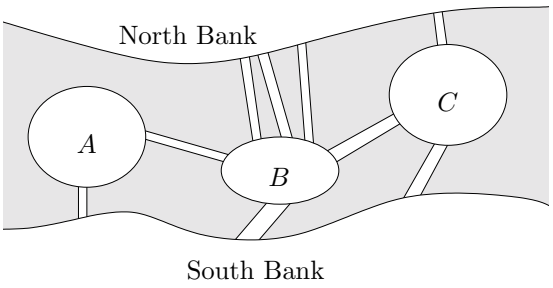
Review for Test on Graph Stuff

Problems 1 through 5 refer to the following graph:



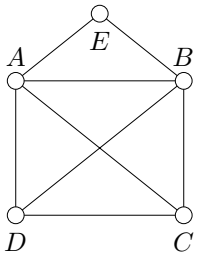
1. Which vertices is vertex A adjacent to?
2. The degree of vertex C is _____.
3. Find three paths from vertex D to vertex A.
4. Find a circuit in the graph.
5. Identify all bridges in the graph.
6. Know Euler's theorems.
7. Know the basic rule in Fleury's algorithm.

Questions 8 through 10 refer to the following situation: A river runs through the middle of a certain city. There are three islands and nine bridges as shown in the figure.

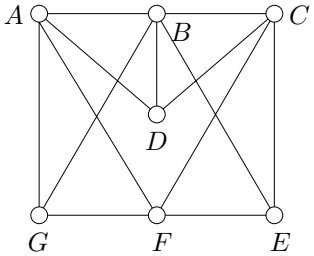


8. A graph that appropriately models this situation would have _____ vertices and _____ edges.
9. It is possible to take a walk through this town, starting on the South Bank, crossing each bridge once (and only once) and ending _____.
10. Suppose that there is a crossing charge of \$1.00 every time one crosses a bridge. A tourist wants to start on the South Bank, stroll across each of the bridges at least once, and return to her hotel on the South Bank at the end of the trip. What is the cheapest possible cost of such a trip?

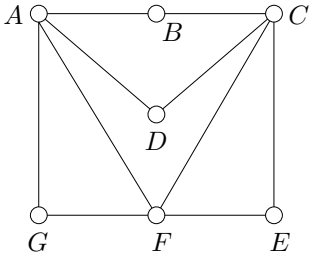
Questions 11 and 12 refer to the following three graphs.



Graph 1



Graph 2



Graph 3

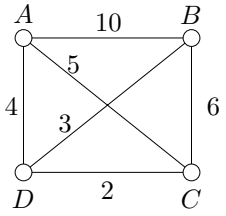
11. Which of the above graphs has an Euler circuit?

12. Which of the above graphs has an Euler path but no Euler circuit?

13. The number of edges in K_{11} is _____.

14. The number of Hamilton circuits in K_{11} is _____.

Questions 15 through 18 refer to the following situation: A delivery truck must deliver furniture to four different locations (A, B, C, and D). The trip must start and end at A. The graph below shows the distances between locations (in miles). We want to minimize the total distance traveled.



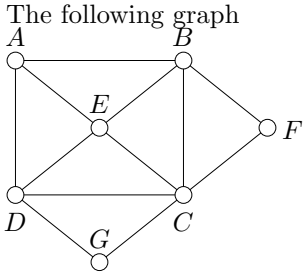
15. What solution does the nearest neighbor algorithm yield?

16. What solution does the cheapest link algorithm yield?

17. What solution does the repetitive nearest neighbor algorithm yield?

18. What is the optimal solution?

19. The following graph



(A) has no Hamilton circuit.

(B) has a single Hamilton circuit (and its mirror image).

(C) has several Hamilton circuits, none of which contain the edge BC.

(D) has several Hamilton circuits, none of which contain the edge AD.

20. Know what an *optimal* algorithm is and what an *efficient* algorithm is.

Problems 21 and 22 refer to the following situation: A traveling salesman's territory consists of the five cities shown on the following mileage chart. The salesman must organize a round trip that starts and ends at Louisville (his hometown) and passes through each of the four cities exactly once.

	Boston	Buffalo	Chicago	Columbus	Louisville
Boston	*	446	963	735	941
Buffalo	446	*	522	326	532
Chicago	963	522	*	308	292
Columbus	735	326	308	*	209
Louisville	941	532	292	209	*

21. What solution does the nearest neighbor algorithm yield?

22. What solution does the cheapest link algorithm yield?

23. Consider the brute-force algorithm for solving the Traveling Salesman Problem. Is it efficient? Optimal?

24. Consider the cheapest link algorithm for solving the Traveling Salesman Problem. Is it efficient? Optimal?