

- Know the definitions of the following:
 - “a player receives their fair share”
 - a violation of the quota rule (lower or upper)
 - the Alabama paradox
 - the population paradox
 - the new states paradox.
- Be able to apply the following methods:
 - divider/chooser
 - lone divider
 - last diminisher
 - sealed bids
 - markers
 - Hamilton’s method of apportionment
 - Jefferson’s method of apportionment
 - Adams’ method of apportionment
 - Webster’s method of apportionment

Questions 3 through 5 refer to the following situation: Three players (one divider and two choosers) are going to divide a cake fairly using the lone divider method. The divider cuts the cake into three slices (s_1 , s_2 , and s_3).

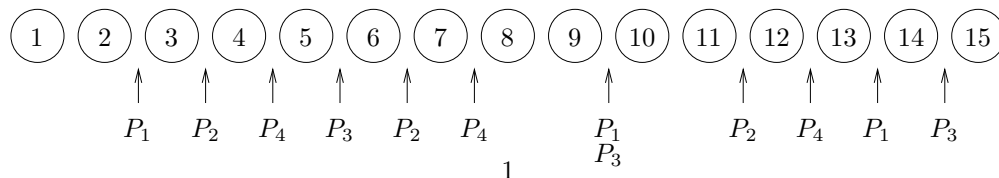
- If the choosers’ declarations are Chooser 1: $\{s_1\}$ and Chooser 2: $\{s_3\}$, describe a fair division of the cake.
- If the choosers’ declarations are Chooser 1: $\{s_2, s_3\}$ and Chooser 2: $\{s_1, s_3\}$, describe a fair division of the cake.
- If the choosers’ declarations are Chooser 1: $\{s_2\}$ and Chooser 2: $\{s_2\}$, describe a fair division of the cake.

Questions 6 and 7 refer to the following example: Three heirs (A , B , and C) must divide fairly an estate consisting of two items (a house and a boat) using the method of sealed bids. The players’ bids (in dollars) are:

	A	B	C
House	180,000	190,000	200,000
Boat	42,000	50,000	31,000

- The original fair share of player C is worth:
- After the initial allocations to each player are made there is a surplus of:

Questions 8 through 10 refer to the following: Four players (P_1 , P_2 , P_3 , and P_4) agree to divide the 15 items shown below by lining them up in order and using the method of markers. The players’ bids are as indicated:



8. Who gets item 3?
9. Who gets item 5?
10. Who gets item 10?

Questions 11 through 13 refer to the following situation: Five players agree to divide a cake fairly using the last diminisher method. The players play in the following order: Anne first, Betty second, Cindy third, Doris fourth, and Ellen last. Suppose that there are no diminishers in round 1 and Cindy and Doris are the only diminishers in round 2.

11. Which player gets her fair share at the end of round 1?
12. Which player is the first to cut the cake at the beginning of the second round?
13. Which player gets her fair share at the end of round 2?

Questions 14 through 22 refer to a country with five states. There are 240 seats in the legislature and the populations of the states are given in the table below.

State	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>
Population	427,000	754,000	4,389,000	3,873,000	157,000

14. The standard divisor is:

The choices for problems 15 through 18 are listed below question 18:

15. The standard quotas (to three decimal places) are:
16. Using a divisor of $D = 40,300$ the modified quotas (to 3 decimal places) are:
17. Using a divisor of $D = 39,540$ the modified quotas (to 3 decimal places) are:
18. Using a divisor of $D = 40,100$ the modified quotas (to 3 decimal places) are:

$A : 10.648; B : 18.803; C : 109.451; D : 96.584; E : 3.915.$

$A : 10.675; B : 18.850; C : 109.725; D : 96.825; E : 3.925.$

$A : 10.799; B : 19.069; C : 111.002; D : 97.951; E : 3.971.$

$A : 10.596; B : 18.710; C : 108.908; D : 96.104; E : 3.896.$

The choices for problems 19 through 22 are listed below question 22:

19. Under Hamilton's method the apportionments to each state are:
20. Under Jefferson's method the apportionments to each state are:
21. Under Adams' method the apportionments to each state are:
22. Under Webster's method the apportionments to each state are

$A : 10; B : 19; C : 111; D : 97; E : 3.$

$A : 11; B : 19; C : 109; D : 97; E : 4.$

$A : 10; B : 18; C : 110; D : 98; E : 4.$

$A : 10; B : 19; C : 110; D : 97; E : 4.$