

The lists of exercises below are subject to change. You can also find true/false questions at the textbook author's website. (This doesn't work with my browser, but maybe it does with yours.) <http://www.d.umn.edu/~jgallian>

CHAPTER 8: DIRECT PRODUCTS

You can multiply two (or more) groups to get a new group.

Exercises: 3,4,6,10,11,14,21,23,49.

CHAPTER 9: NORMAL SUBGROUPS; FACTOR GROUPS

Recall that when H is a subgroup of G , we have an associated collection G/H of *left cosets* of H in G , and a collection $H\backslash G$ of *right cosets*. Under certain circumstances, these collections are equal, and moreover form a group. (To put it another way: in the previous chapter, we learned to multiply groups to get new groups. Now we're learning to divide groups.)

Exercises: 1,4,5,7,10,35,36,44,62.

CHAPTER 10: HOMOMORPHISMS

...like isomorphisms, but not necessarily one-to-one or onto.

Exercises: 1–3, 7–9, 15(you *don't* need to determine ϕ), 37, 38.

CHAPTER 11: LET'S FIND ALL FINITE ABELIAN GROUPS

It's not hard to list, say, all abelian groups of order 150. Surprised?

Exercises: 1,2,5,9,15,16,17,24. (Hint on 24: See example 1 and exercise 23.)

CHAPTER 15: RING HOMOMORPHISMS (AGAIN)

We'll do the bits we skipped last semester.

Exercises: 1(part 3),2,11,31(see #30),35,36,43,45,59.

CHAPTER 24: SYLOW'S THEOREMS

To quote from Herstein's *Topics in Algebra*, "I love Sylow's Theorem." Because of other goals of the course, we don't have time to cover this in full. But the result is so fun to apply that I can't resist at least stating it. Why should you miss out?

Exercises: 1,3,11,12,13,20,25.

(over)

CHAPTER 17: FACTORIZATION OF POLYNOMIALS**Exercises:** 6,10,11,13

CHAPTER 19: VECTOR SPACES

Now we head toward Galois theory, the main goal of the course. In order to do so, we must cover (or re-cover) the basics of linear algebra.

Exercises: 1(example 2),4,7,9,10,14,19,28,29.

CHAPTER 20: EXTENSION FIELDS

Let $f(x) = x^2 + 1$. Recall that $f(x)$ has no root in \mathbb{R} . But at some point in your life, you learned that there's a larger number system, namely \mathbb{C} , where $f(x)$ has roots. More generally, we'll see that given any field F and any polynomial $f(x) \in F[x]$ with no roots in F , we can always construct a bigger field E where $f(x)$ has roots.

Exercises: 2,3,7,8,10,13,20,26,30,34,35.

CHAPTER 21: ALGEBRAIC EXTENSIONS**Exercises:** 1(hint on 21.3: division algorithm),3,7–9,14,23,26,29,35.

CHAPTER 22: FINITE FIELDS

Since these fields are quite unlike the more familiar fields \mathbb{Q} and \mathbb{R} , they form a good test-bed for field theory. Moreover, they turn out to be useful in their own right.

Exercises: 1,7,10(hint: try 9),17,19,21,24,27,31.

CHAPTER 23: GEOMETRIC CONSTRUCTIONS

Several of the great problems of antiquity involve ruler-and-compass constructions. Examples: "Trisect an angle." "Square the circle." People tried for many centuries to realize these constructions. Finally, the early field theorists were able to show that they are impossible. Does "impossible" mean simply that no one has ever found a way to do them? No, it means that they really are impossible.

Exercises: 1–4,13–17.

CHAPTER 32: GALOIS THEORY

This is the original motivation for the invention of group theory. See another great problem of antiquity solved.

Exercises: 1–3,5,23.

Galois theory makes a fitting conclusion to the course. But if we have extra time, we'll look at a striking application: a proof of the Fundamental Theorem of Algebra that makes efficient use of both Sylow's theorem and Galois theory.
