Transcript of the panel discussion.

The moderator: A. Friedman, Regents' Professor, University of Minnesota.
Panelists: G. Allaire, Professor of Mathematics, Ecole Polytechnique, Paris;
R. Lakes, Wisconsin Distinguished Professor, University of Wisconsin-Madison;
G. Milton, Distinguisged Professor, University of Utah

We briefly summarize the main points of the panel discussion.

The moderator, A. Friedman, suggested the following issues and questions for the discussion.

Issue 1. There is a large community of materials science, and there is also a large mathematical homogenization community. There is an intersection between the two and there are questions that are of interest for both communities. Even though homogenization as a technique is mathematically very powerful, there is still a big gap between the homogenization techniques and practical needs. On the one hand, standard homogenization methods give precise formulas but make a priori assumptions on the geometry or other structural components of a composite material. In many practical situations these assumptions are not physically valid. On the other hand, estimates on the effective properties of composites are usually not sufficiently precise.

In this connection the following questions arise:

1.What are the prospects for closing this gap? This is one of the most important questions for continued success of the theory.

2.Where are new ideas going to come from?

3.What is the relationship between the mathematical results and the actual materials?

Issue 2. Name some challenges in specific areas of application e.g. porous media, production of steel, biomaterials, etc.
Can we extend our techniques to other types of materials, in particular, biomaterials?

Panelist 1 (G. Allaire).

I would like to describe some future directions in homogenization. I will distinguish between theoretical and numerical aspects.

Theoretical aspects

1. We need to study nonlinear problems. Some tools have been developed. Young measures and defect measures have been introduced and they have been successful in many applications. But there are still many completely open nonlinear problems.

2. Another area of interest is homogenization of evolution problems. By now we know when the homogenization of an evolution problem is more or less equivalent to the homogenization of the corresponding static problem. However, there are many open questions for problems with hysteresis or problems involving evolution of microstructure.

3. There are old problems, which are not yet solved. We have seen, for example, this morning, the problem of boundary layers. So far the boundary layers were studied for flat domains and rectangular boundaries. There is a need to study the boundary layer problems for general domains

4. Another area is singular perturbation problems coupled with homogenization. Here the issue is how to handle one small parameter versus many small parameters, particularly when these small parameters are dependent upon each other.

5. Homogenization of eigenvalue problems, spectral homogenization, is not well understood. This is linked to singular perturbation problems, such as those examining the high frequency regime.

Numerical aspects.

1. In optimal design there are new problems that can be attacked with homogenization both analytically and numerically.

2. Homogenization of non-periodic problems using oscillating test functions and wavelets in finite element analyses is one possible way of bridging the gap between the periodic assumption and the non-periodic physical situation.

3. Specific challenges: homogenization of two-phase flow, water and steam, for example. There is still no universal numerical model of two-phase flow.

4. Homogenization of reactor physics is another tough problem. We do not have enough computer power for computations at the macroscale in these problems.

The moderator: I would like to reiterate two points. First, the use of homogenization techniques for non-periodic problems by the use of oscillating test functions. Second the modeling of two-phase flows: we had two workshops at the IMA in Minnesota during the past 10 years, but the problem is still unsolved. No one knows how to average the product.

Panelist 2 (R. Lakes).

Among the opportunities I see is treatment of biological materials (with biomagnetics). This is a very active area. The biological materials are very subtle in structure, refined and sometimes surprising in their behavior. Some people are trying to imitate unusual properties of bio-materials. This type of composite theory is much more difficult to develop because the constituent materials are not known. At present our knowledge of the subject is very limited and homogenization theory predictions can guide experiments.

Another interesting area is nano-composites (very fine microstructure). Here chemical physics and chemistry play significant roles. The key issue is how to handle the interface between materials. This would require new tools and new insights.

Panelist 3 (G. Milton).

1. Identification of important parameters. We are interested in the effective conductivity σ^* of a composite. Can one find a few parameters $\alpha_j, j = 1, 2, 3...$ that are easy to measure, and σ^* is strongly correlated with α_j? It would help to get tight bounds on σ^* incorporating α_j. For example, one can choose the following parameters:
α_1-volume fraction of phase 1;
α_2-conductivity when $\sigma_1 = 0$ or ∞;
α_3-????

2. Correction to homogenization.
What happens when ϵ is small but not zero? Probably, non-local effects should be important. When inhomogeneities of several scales are present, an intermediate description should clearly allow some inhomogeneities to be retained. The intermediate description should thus have both non-local effects and inhomogeneities present. Most intermediate descriptions have one or the other but not both.

3. Coarse Graining: continuous homogenization.
Is there some intermediate description, which allows one to continuously progress from an inhomogeneous media to a homogeneous effective medium, with the inclusions becoming more and more blurred, until everything ultimately becomes homogeneous.

4. Other outstanding problems.
a. Wave propagation in inhomogeneous random media when the wavelength is comparable with the microstructure scale;
b. Nonlinear media;
c. Homogenization at the boundary of ellipticity (degenerate elliptic problems) e.g. when the materials properties are of order ϵ^2

Summary of the discussion and comments from the audience on important directions in homogenization theory and questions to be addressed.

Homogenization of random media (statistical homogenization) is an open area. General existence type results are available but very few results have been used in practical applications. There is a need to develop many more efficient tools (formulas) to describe random media (disordered materials).

Is it possible to achieve better extreme properties with random microstructure as compared with deterministic microstructure?

Fracture of heterogeneous materials is poorly understood. Molecular and static fracture mechanics are completely different.

Fracture goes along grain boundaries. Homogenization may not be a good tool here.

Homogenization always goes from micro to macro. How about the reverse?

Accuracy of various homogenization formulas is an important practical issue.

An interesting question. If you have a material, what are its properties in an external field? For instance, particles of rocket fuel can explode in intense fields.