Probability of Independent Events
Demonstrating that $P(A \cap B) = P(A) \cdot P(B)$

Prior Knowledge: Probability Tree, Sample Space, Finding simple probability, Fractions

Activity

This is a probability tree for choosing two pieces of pizza from the above picture.

1. What is the probability that your first piece is mushroom? ______________

2. What is the probability that your second choice is pepperoni given that your first choice was mushroom? ______________

3. Count in the given sample space. How many times will your first choice be mushroom and your second choice be pepperoni? ______________
4. What is the probability that your first choice will be mushroom and your second choice be pepperoni? __________________________________________________________

5. Use your answers from exercises 1, 3, and 4 to fill in the following:

\[ P(\text{M } 1^{\text{st}}) = \] __________
\[ P(\text{P } 2^{\text{nd}} \text{ after M}) = \] __________
\[ P(\text{M } 1^{\text{st}} \text{ and then p } 2^{\text{nd}}) = \] __________

What did you notice about these fractions? ________________________________________________

\[ \begin{array}{c|c|c|c|c|c|c}
\hline
& c & c & m & m & m & m \\
& c & c & m & m & m & m \\
& p & p & a & a & a & a \\
& p & p & a & a & a & a \\
\hline
\end{array} \]

c = cheese, p = pepperoni, m = mushroom, a = anchovy

6. For this new pizza, make a probability tree for 2 choices.
7. What is the P(M 1st)?

8. What is P(C 2nd given M 1st)?

9. List the Sample Space.

10. How many different possibilities?

11. Using the sample space, count the number of outcomes are mushroom 1st, then cheese 2nd.

Use the sample space to answer the following.

12. P(A 1st)

13. P(P 2nd)

14. P(A 1st and then M 2nd)

15. P(M 1st)

16. P(A 2nd)
17. P (M 1\textsuperscript{st} and then A 2\textsuperscript{nd})

Now, list your answers from exercises 7, 8, 12-17 below.

\begin{align*}
P(M) & \quad P(A) \quad P(M) \\
P(C) & \quad P(P) \quad P(A) \\
P(M \text{ and then } C) & \quad P(A \text{ and then } P) \quad P(M \text{ and then } A)
\end{align*}

Think back to exercise # 5. What do you notice about these fractions? __________

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18. Make a conjecture about the probability of one event then another event occurring. __________

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19. Make a pizza of your own with at least 5 different pieces. Draw a probability tree and label each branch with the probabilities of the individual choices. Then make a column to the right of the tree and fill it in with the sample space and another column for the probability of each event. (See #1 for an example)

20. Refer to the conjectures that you made in exercises 5 and 18. If you have changed your mind, make adjustments here. ________________

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