Circumference
Inquiry-based lesson—teacher lesson plan

Summary: This lesson is designed to lead the students through finding the ratio between diameter and circumference of a circle. The students will call the constant ratio “pi” (π) and use it to find a formula for the circumference of a circle. They will also work on expressions involving π and the measures of a circle.

Key Words: circle, circumference, diameter

Background Knowledge: multiplication, use of a ruler and reading it correctly, metric measurements, unit conversions, variables

NCTM Standards Addressed: In the Geometry Standard it states “Use geometric models to represent and explain numerical and algebraic relationships,” students will accomplish this by shifting from the visual perception of circles and the different measures on them to the algebraic expression involving π that gives the exact circumference or diameter.

Learning Objectives: Students will discover the ratio of π, formulate the equation of the circumference of a circle, and support their hypothesis with examples.

Materials: various round objects (frisbee, clock, paper plate, plastic lids, cookies, coins, cups), tape measure or string and rulers, calculators

Suggested Procedures: (Set Induction) Tell a story about rolling a flat tire down the road a certain distance to a service station. (Grouping) Teacher’s preference.

Assessments: Observation during work time in class, completing extension problems.
Circumference

Team members’ names ____________________________________________________________

Goals:
Discover the ratio of “pi” (π), formulate the equation of the circumference of a circle, and support your hypothesis with examples.

Investigation:
Using the tape measure or string and ruler, measure the diameter and circumference of each object provided. The diameter is the distance across a circle at its widest point and the circumference is the distance around the outside of a circle. Use metric measurements and organize them in the table provided below. Then find the ratio of circumference to the diameter by dividing the value obtained for circumference by the value obtained for diameter.

<table>
<thead>
<tr>
<th>Object</th>
<th>Diameter</th>
<th>Circumference</th>
<th>Ratio</th>
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</tbody>
</table>

Do you notice any patterns in the table? If so, what are they? ______________________

Estimate the circumference of a tire with a diameter of 76 cm. ______________________

How was this answer found? ______________________________________________________

The ratio of circumference of a circle to diameter of a circle is a constant called “pi”. This is represented with the Greek letter π.
What is the approximate value of $\pi$? 

On your calculator find the button labeled with $\pi$ and press it. Now give a more exact value of $\pi$. 

Which is a closer approximation for $\pi$, 3.14 or 22/7? Explain. 

Write a formula that gives the circumference of a circle in terms of the diameter of the circle and $\pi$. 

Using two more round objects of your choice, test your formula by measuring the diameter, calculating the circumference with the formula, and then measuring the circumference to check the calculation. In the last column, compare the difference between the calculated and measured circumference. Record your results on the table below.

<table>
<thead>
<tr>
<th>Object</th>
<th>Diameter</th>
<th>Calculated Circumference</th>
<th>Measured Circumference</th>
<th>Comparison</th>
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</thead>
<tbody>
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</tbody>
</table>

Do you think your formula is correct? Why or why not? 

* If your formula was not correct, fix it!!!

Find the circumference of a circle that has diameter 20 cm. 

Find a more exact value for the circumference of the tire that had diameter 76 cm. How does this compare to the earlier result?
Extension problems:

1. Suppose you have a car with tire diameter of 76 cm. How far would the tire roll in one complete turn? In 10 complete turns? ________________________________

2. About how many complete turns would it take to roll the tire 400 meters (1 meter = 100 centimeters) down the road to a service station? How do you know? _____

3. Find the diameter of a clock which has a circumference of 120 cm. ____________

4. Choose one of the round objects that will be easy to roll.

   a. Measure the diameter of the object and then use the formula to find the circumference. Record these values.______________________________

   b. Mark a starting line on the floor and continuously roll the object several times, then measure how far it rolled and record that distance.__________

   c. How many complete turns did the object make? How do you know?_____

Circumference
Inquiry-based lesson—solutions

Team members’ names

Goals:
Discover the ratio of “pi” (π), formulate the equation of the circumference of a circle, and support your hypothesis with examples.

Investigation:
Using the tape measure or string and ruler, measure the diameter and circumference of each object provided. The diameter is the distance across a circle through the center and the circumference is the distance around the circle. Use metric measurements and organize them in the table provided below. Then find the ratio of circumference to the diameter by dividing the value obtained for circumference by the value obtained for diameter.

<table>
<thead>
<tr>
<th>Object</th>
<th>Diameter</th>
<th>Circumference</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>lid</td>
<td>11 cm</td>
<td>34 cm</td>
<td>3.1 cm</td>
</tr>
<tr>
<td>frisbee</td>
<td>14 cm</td>
<td>43 cm</td>
<td>3.1 cm</td>
</tr>
</tbody>
</table>

Do you notice any patterns in the table? If so, what are they? Yes, we notice a pattern. The ratio is approximately 3.1 cm each time.

What does this mean? The circumference of a circular object is approximately 3.1 times the diameter.

Estimate the circumference of a tire with a diameter of 76 cm. 235.6 cm

How was this answer found? We multiplied 76 by 3.1 and found 235.6 cm.

The ratio of circumference of a circle to the diameter of a circle is a constant called “pi”. This is represented with the Greek letter π.
What is the approximate value of π? 3.1

On your calculator find the button labeled with π and press it. Now give a more exact value of π? 3.141592654

Which is a closer approximation for π, 3.14 or 22/7? Explain. 22/7 because if you round π to 3 decimal places you would get 3.142 which matches 22/7.

Write a formula that gives the circumference of a circle, C in terms of the diameter of the circle, D and π. C=3.14*D or C=πD
Using two more round objects of your choice, test to see if the formula you chose for the circumference of a circle is correct. Start by measuring and recording the diameter of your object. Next, calculate the circumference of the object with your formula, and then measure the actual circumference of the object and record it. In the last column, compare the difference between the calculated and measured circumference. Record your results on the table below.

<table>
<thead>
<tr>
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<th>Diameter</th>
<th>Calculated Circumference</th>
<th>Measured Circumference</th>
<th>Comparison</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plastic cup</td>
<td>5.5 cm</td>
<td>17.28 cm</td>
<td>17.4 cm</td>
<td>Off by .12 cm</td>
</tr>
</tbody>
</table>

Do you think your formula is correct? Why or why not? Yes, since our comparison was so close.
* If your formula was not correct, fix it!!!

Find the circumference of a circle that has diameter 20 cm. 62.83 cm

Find a more exact value for the circumference of the tire that had diameter 76 cm. How does this compare to the earlier result? 238.76 cm. Earlier we got 235.6 cm and that is short by 3.16 cm.

Extension problems:

5. Suppose you have a car with tire diameter of 76 cm. How far would the tire roll in one complete turn? In 10 complete turns? 238.76 cm in one turn, 2,387.6 cm in 10 complete turns.

6. About how many complete turns would it take to roll the tire 400 meters (1 meter = 100 centimeters) down the road to a service station? How do you know? 167.53 complete turns. We divided 40,000 by 238.76 cm.

7. Find the diameter of a clock which has a circumference of 120 cm. 38 cm.

8. Choose one of the round objects that will be easy to roll.
   a. Measure the diameter of the object and then use the formula to find the circumference. Record these values. The diameter of the frisbee is 14 cm and the circumference is 43.98 cm.
   b. Mark a starting line on the floor and continuously roll the object several times, then measure how far it rolled and record that distance. 3.2 m equals 320 cm.
   c. How many complete turns did the object make? How do you know? 7 complete turns (7.28 turns). We divided 320 by 43.98 cm and also counted as the frisbee turned. (marked a point of reference on the frisbee).