FRACTALS ROCK: INVESTIGATING THE SIERPINSKI TRIANGLE

LESSON PLAN

Fractals: A figure generated by repeating a special sequence of steps infinitely often. Fractals often exhibit self-similarity (the fractals clone themselves).

Introduction: This is another example of a fractal curve. Instead of using one that builds outwards, this construction starts with a triangle and builds inwards. This particular fractal construction is called a Sierpinski Triangle (Gasket). It, like the other fractal curves, has its own unique characteristics and subsequent behavior as the number of iterations increases.

Strand: Non-Euclidean Geometry - Fractal

Learning Objectives:

1. Students will be able to construct a Sierpinski Triangle (Gasket) by using a macro within Cabri Geometry II.

2. Students will be able to make conjectures about the behavior of the fractal curve as the number of iterations increases by providing charts and graphs as support. This might include what happens to the perimeter, area, or total number of triangles.

3. Students will be able to generalize the results in the form of an equation that will explain the observed behavior as the number of iterations goes up.

Materials: Cabri Geometry II, Lesson Labs for each student to use in the group. (Alternative: This lesson can be done using TI-92 or TI-92+ calculators)

Procedure: (suggestions)

1. Set Induction/Attention getter: Review with the students that other fractals involved fractal curves that were constructed “outward” from the original. Ask the students if a similar thing could be done inward.

2. Group students using method of choice.

3. Distribute lesson labs and allow plenty of time to complete them.
3. Bring the groups back into a whole class discussion. Discuss the characteristics that each group has observe. Generalize these observations, and allow the students to compare and contrast the characteristics of each of the three fractal curves they have studied.

**Assessment:**

Authentic forms of assessment are usually best when conducting a lesson that relies heavily on discovery based learning. Keep this in mind when assessing the students. The possibilities for assessment are left up to the instructor.
Lab: Creating a Sierpinski Triangle

Setup:
Select the **Pointer Tool** and using the option buttons color to dark green and the point size to the largest solid value.
Select the **Segment Tool** and using the option buttons set the color to purple and the line size to the smallest value.
Select the **Polygon Tool** and using the option buttons set the color to light green and the line size to the middle value.

A: **Create a macro that generates internal triangles.**

1. Construct an equilateral triangle. (Regular Polygon Tool) Label the vertices A, B, and C.
2. Select this triangle as the initial object for a macro.
3. Construct the Midpoints of each side of the triangle. Label them DEF.
4. Construct the three triangles formed by each vertex and its adjacent midpoints. (Polygon Tool) Your figure should look like the figure at right.
5. Select the three new little triangles as the final objects for your macro.
6. Define the Macro as Sierpinski 1.

B. **Create a macro that generates a second iteration Sierpinski Triangle.**

1. Clean the screen
2. Construct an equilateral triangle (Regular Polygon Tool).
3. Select this triangle as an initial object for a new macro.
4. Use the Sierpinski 1 macro to create a first iteration Sierpinski Triangle.
5. Use the Sierpinski 1 macro to create a second iteration Sierpinski Triangle by clicking on each of the lines joining the midpoints. Your figure should appear as at right.
6. Select each of the final little triangles as the final objects for a new macro.
7. Define the macro as Sierpinski 2.

8. Describe in your own words what each subsequent iteration does to the pre-existing figure.

9. If you were to keep performing iteration after iteration, what would happen to the number of newly formed triangles? Explain.

10. As the number of iterations approaches infinity, what happens to the area of each newly formed triangle? Explain.

11. If the smallest triangle of each iteration were shaded, what would the total area of the shaded area be after each iteration?

12. Calculate the perimeter of each of the newly formed triangles. What happens to the perimeter of each of the newly formed triangles as the number of iterations increases? Explain.

Explorations:

1. Using the Sierpinski Triangle macros, how many iterations can you perform and still see the smallest triangles if the first triangle is as large as possible to still fit on the screen?

2. What type of starting triangles, besides equilateral, can be used to generate Sierpinski Triangles?

Extensions:

1. Create a Sierpinski triangle whose iterations is not based on triangles joining the midpoints of the sides of the original triangle?

2. Can you create a “Sierpinski Square? How about other regular polygons? How about other irregular polygons?