FRACTALS ROCK:
Investigating The Fractal Tree

Keyword:
Fractal (Fractal Tree)
Macros

Fractals:
A figure generated by repeating a special sequence of steps infinitely often. Fractals often exhibit self-similarity (the fractals clone themselves).

Introduction:
Constructing a Fractal Tree

In this lesson, the students will learn how to construct a fractal tree. Fields such as computer science have grown to new heights as a result of fractal geometry. Fractal curves become somewhat complex once they have gone through a number of iterations. The use of macros in computer programs such as Cabri, however, make the task of constructing these curves quite simple.

Strand: Non-Euclidean Geometry - Fractal

Learning Objectives:

1. Students will be able to construct a fractal tree and explore variations to the original construction using a macro within Cabri Geometry II.

2. Students will be able to identify patterns when the number of iterations on the fractal tree increase using charts or tables.

3. Students will be able to hypothesize the general behavior of the fractal tree as the number of iterations increases i.e. giving the equation for what happens to the number of segments with each iteration?

Materials: Cabri Geometry II, Lesson Labs for each student to use in the group. (Alternative: This lesson and lab can also be done with TI-92 or TI-92+ calculators.)
Procedure: (suggestions)

1. Set induction/Attention getter: Ask the students what coastlines, mountain ranges, snowflakes, and a DNA cluster have in common. Allow a few students to answer. Tell them that all of these things can be generalized using a type of geometric curve called a fractal curve. Ask if anyone has ever heard of such a curve. Let the discussion lead into the lesson.

2. Group students using method of choice.

3. Distribute lesson labs and provide plenty of time to complete them. One might include the methods for constructing macros as part of the lesson.

4. Conclude with a class discussion summarizing what the students observed during the lesson.

Assessment:

Authentic forms of assessment are usually best when conducting a lesson that relies heavily on discovery based learning. Keep this in mind when assessing the students. The possibilities for assessment are left up to the instructor.
Fractals Rock:
Dynamic Geometry Lab – Creating the Fractal Tree

Team Members: ____________________
________________________________

File Name: ____________________

Setup:
Select the **Pointer Tool** and using the option buttons color to dark green and the point size to the largest solid value.
Select the **Segment Tool** and using the option buttons set the color to purple and the line size to the smallest value.
Select the **Polygon Tool** and using the option buttons set the color to light green and the line size to the middle value.

A. Construct a macro to tri-sect a line and put a perpendicular segment on the 1/3 point that is 2/3 the length of the original segment.

1. Construct a horizontal line, \( l \), and construct points \( A \) and \( B \) on it. \((\text{Line and Point Tools})\).

2. Construct \( AB \). \((\text{Segment Tool})\)

3. Make \( AB \) the initial object in a new macro. \((\text{initial object in Macro Tool})\).

4. Construct a line, \( m \), through point \( A \) that is at an acute counterclockwise angle from \( AB \) \((\text{Line Tool})\)

5. Draw a circle centered at \( A \) of arbitrary but reasonably small size (see figure above).

6. Draw two more circles centered at the intersection of the previous circle and the line \( m \) (see figure at right).
7. Construct the line from the intersection of line m and the last circle to point B. label this line n. Construct a line, o, parallel to n through the intersection of line m and the first circle. Construct the point at the intersection of line o and \( \overline{AB} \) (see figure below).

![Diagram showing the construction steps]

8. Hide the three circles and lines n and o.

9. Construct a line through P perpendicular to \( \overline{AB} \).

10. Construct a circle centered at P with radius \( \overline{AP} \).

11. Construct points C and D at the intersection of the circle centered at P with the perpendicular line through P.

12. Construct \( \overline{AP} \), \( \overline{BP} \), \( \overline{CP} \), and \( \overline{DP} \). The order of construction is important. Always finish the segment at point P.

13. Hide everything except the four segments: \( \overline{AP} \), \( \overline{BP} \), \( \overline{CP} \), and \( \overline{DP} \). Select these segments as the final objects for the macro.

14. Define the Macro to be Fractal Tree 1.

B. Constructing a Fractal Tree.

1. Clear the screen. Construct \( \overline{AB} \). Set this segment as the initial object for a new macro.

2. Select the Macro Fractal Tree 1. Click on the segment.. Then click on each of the small segments. Does your picture should look like the one at the right?

![Diagram showing the construction steps]

3. Select all the segments as final objects for the new macro. (All the lines should be blinking). Define the Macro as Fractal Tree 2.
4. Clear the screen. Construct a new $\overline{AB}$. Select Fractal Tree 2 macro. Click on the segment.

5. Now click on each of the small branches. This is a fourth level tree fractal. Does yours look like the one below? Grab point B and move it around? What happens?

Extensions:

1. How many levels can you generate and still see the detail of the tree (keeping all the branches on the screen)?

2. Construct a tree fractal with a different branch pattern.