Menelaus’ Theorem

**Lesson Summary:**
The students will be placed in pairs to help them learn to cooperate and help one another through self-discovery and the cooperative activity. The students will work individually on journal prompts. This will help develop creativity and written communication skills.

**Key Words:**
Modern geometry, Menelaus’ Theorem

**Background Knowledge:**
- Students will have prior lessons on polygons and area.
- Students will have prior experience in recognizing patterns.
- Students must have some prior geometry and problem solving skills.

This information helps the teacher to plan the daily lesson prior to this activity to ensure that students have this prior knowledge. It will be important to know that the students can recognize patterns.

**Learning Objectives:**
- Students will discover the relationship between the boundary points, interior points, and area of lattice point polygons – Pick’s Theorem.
- Self discovery will help the students become self-learners.
- Students will collaborate with others.

**Materials:**
This lesson can be done one of two ways. The students can either work together on Cabri Geometry II software on computers or calculators. Or the students can use Geo Boards.
- Overhead to give students information
- Geo Boards or Computers/Calculators equipped with Cabri Geometry II.
- Lab Handout
- Journal Prompt

**Suggested Procedure:**
This lesson is designed for a Math class consisting of 10th and 11th grade students in Geometry. The time allocated will be one day with a take home assignment.

**Grouping:**
Students will be put in pairs for the self discovery and cooperative activity. They will take notes and observe individually during whole group instruction. Students will also have an individual take home journal prompt.

**Students with Special Needs:**
Special Needs students will be grouped with others and will be able to participate in group activities and peer collaboration. Available equipment will be used if students are visually or hearing impaired. Low level students may be given modified questions to answer in addition to group
activity to access their level of understanding if different than other group members.

1. **Introduction**
   - Quick Set Induction to gain student interest.

2. **Group activity**
   - Students will get into pairs.
   - The pairs group will work together to construct various polygons either on a Geo Board or Cabri.
   - Using the constructions, the pairs will complete a table and answer various questions.
   - The teacher will monitor the groups to ensure all group members are participating and to assist with any questions.

1. **Conclusion**
   - Class discussion of today’s discoveries
   - Class summary and journal assignment will conclude the lesson.

**Assessment:**
- Possible points for class and group participation will be 10. (This will take the place of a quiz score). Teacher will observe students to ensure that each student participates and assists other group members.

- Possible points for journal prompt will be 10 also.

- Each student will be given a review handout in two days that will also cover this material and it will be worth 30 points.
Theorem of Menelaus

Team members' names: ______________________________________________________

File name: ______________________________________________________________

**Goal:** After completing this lab the students should have discovered Menelaus’ Theorem.

**Procedure:** (Use Cabri*)

**Part A:** Constructing the diagram

1. Construct a triangle \( \Delta ABC \) such that segment \( BC \) is the base of the triangle and label the vertices. [Use triangle tool and label tool]

2. Draw a line through the base of the triangle \( \Delta ABC \) and label this line \( m \). [Use the line tool and the label tool]

3. Draw a line \( n \) through side \( AB \) and side \( AC \) and intersecting line \( m \) [Use the line tool and label tool]

4. Label the intersection of \( n \) and side \( AB \) with \( G \), the intersection of \( n \) and side \( AC \) with \( H \) and the intersection of line \( n \) and line \( m \) with \( I \). [Use the label tool]

5. Create lines \( BF \), \( AD \), and \( CE \) such that these lines are perpendicular to the transversal line \( n \). [Use perpendicular line tool]

6. Construct segments \( BF \), \( AD \), and \( CE \). Hide the perpendicular lines created in (5). [Use segment tool and hide/show tool]

\[
\begin{align*}
\text{m} \angle BFG = \text{m} \angle ADG = \text{m} \angle ADH = \text{m} \angle HEC = \text{m} \angle BFI = \text{m} \angle CIE = 90 \text{ degrees}
\end{align*}
\]
7. Measure segments $\overline{AG}$, $\overline{GB}$, $\overline{CH}$, $\overline{HA}$, $\overline{CE}$ and $\overline{AD}$. Place these values in a blank part of the screen and label them accordingly.

[Use the distance and length tool and label tool]

**Part B:** Consider $\triangle BFG$ and $\triangle ADG$

1. $\angle FGB$ and $\angle AGD$ are what types of angles? ______________________
   This means their measures are ______________________.

2. $\angle GFB$ and $\angle GDA$ are what types of angles? _________________
   This means their measures are ______________________.

3. Considering steps (1) and (2) you now know that $\triangle BFG$ and $\triangle ADG$ are what type of triangles?
   _____________________________________________________________

4. Compute the ratio of segment $\overline{AG}$ to segment $\overline{GB}$.
   [Use the calculate tool]

5. Move this ratio into a blank section of the workspace and label it. Also, change its color to red. [Use comment box and color attributes]

6. Compute the ratio of segment $\overline{AD}$ to segment $\overline{BF}$. [Use the calculate tool]

7. Move this ratio into a blank section of the workspace and label it. Also, change its color to blue. [Use comment box and color attributes]

8. What do you notice about the ratios found in (4) and (6)?
   _____________________________________________________________

9. **Print** and **Save As** *Menelaus1*

**Part C:** Consider $\triangle ADH$ and $\triangle CEH$

1. $\angle ADH$ and $\angle CEH$ are what types of angles? ______________________
   This means their measures are ______________________.

2. $\angle AHD$ and $\angle CHE$ are what types of angles? ______________________
   This means their measures are ______________________.

3. Considering steps (1) and (2) you now know that $\triangle ADH$ and $\triangle CEH$ are what types of triangles? ______________________
4. Compute the ratio of segment $\overline{CH}$ to segment $\overline{HA}$. 
   [Use the calculate tool]

5. Move this ratio into a blank section of the workspace and label it. Also, change its color to red. [Use comment box and color attributes]

6. Compute the ratio of segment $\overline{CE}$ to segment $\overline{AD}$. 
   [Use the calculate tool]

7. Move this ratio into a blank section of the workspace and label it. Also, change its color to blue. [Use comment box and color attributes]

8. What do you notice about the ratios found in (4) and (6)?

   ____________________________________________________________

9. **Print and Save As Menelaus2**

**Part D:** Consider $\triangle FBI$ and $\triangle ECI$

1. $\angle IFB$ and $\angle IEC$ are what types of angles? __________________________
   This means their measures are ________________________________.

2. $\angle FIB$ and $\angle EIC$ are what types of angles? _______________________
   This means their measures are ________________________________.

3. Considering steps (1) and (2) you now know that $\triangle FBI$ and $\triangle ECI$ are what type of triangles? __________________________________________

4. Compute the ratio of segment $\overline{BI}$ to segment $\overline{IC}$. 
   [Use the calculate tool]

5. Move this ratio into a blank section of the workspace and label it. Also, change its color to red. [Use comment box and color attributes]

6. Compute the ratio of segment $\overline{FB}$ to segment $\overline{EC}$. [Use the calculate tool]

7. Move this ratio into a blank section of the workspace and label it. Also, change its color to blue. [Use comment box and color attributes]

8. What do you notice about the ratios found in (4) and (6)?

   ____________________________________________________________

9. **Print and Save As Menelaus3**
**Part E:** Discover the connections between the ratios.

1. Multiply all the red ratios. The product is _______.  
   [Use the calculate tool]

2. Multiply all the blue ratios. The product is _______.  
   [Use the calculate tool]

3. What do you notice about the two ratios? ____________________________

**Part F:** Discover the Menelaus’ Theorem.

1. Using the information from **Part A-F**, what can you say about how a line cutting through two sides of a triangle divides the sides of a triangle?

   ____________________________________________________________________

   ____________________________________________________________________

   ____________________________________________________________________

*If you are not familiar with Cabri’s tools, press F1. A help menu for each tool selected will appear on the screen.*