Lesson I: Triangles- Exterior Angle Theorem

**KEY WORDS:** Triangles, exterior-angle theorem, and remote interior angles.

**Grade Level:** High School

**SUMMARY:**
With this investigation students will discover the relationship between the two remote interior angles (non-adjacent angles) and the exterior angle of a triangle. The exterior-angle theorem states that the measure of each exterior angle of a triangle equals the sum of the measures of its two remote interior angles.

**EXISTING KNOWLEDGE:**
1. It is assumed that students already know and understand the definition of exterior, and remote interior angles.
2. Students will need to understand that the sum of the interior angles of a triangle equals 180 degrees, and supplementary angles also equal 180 degrees.
3. Students will have basic knowledge of how to use Cabri Geometry II.

**STANDARDS:**
1. Formally define geometric figures (i.e. interior and exterior angles) (Geometry and Spatial Sense # 2)
2. Recognize and apply angle relationships in situations involving intersection lines. (Geometry and Spatial Sense # 3)
3. Solve equations and formulas for a specified variable. (Patterns, Functions, and Algebra # 3)

**LEARNING OBJECTIVES:**
1. Students will be able to find the measure of an exterior angle from the two remote interior angles of a triangle.
2. Students will be able to explain the relationship between supplementary angles and the exterior angle theorem.

**MATERIALS:**
Computer lab or set of calculators equipped with Cabri Geometry II and lab worksheets.

**SUGGESTED PROCEDURES:**
- Group students in pairs. The instructor may choose the method to determine the pairing.
- Review a couple of examples using/ explaining the triangle-sum theorem, and supplementary angles.
- Have students work in pairs to perform the following constructions.
Lesson I: Triangles - Exterior Angle Theorem

Team Member's Names: ____________________  
____________________

File Name: ____________________

GOAL 1: Students will be able to find the measure of an exterior angle from the two remote interior angles of a triangle.

INVESTIGATE USING CABRI GEOMETRY II

1. Choose preferences from the options menu. Set angle to 2 decimal places.

2. Construct \( \overrightarrow{AB} \). (use ray tool)

3. Place point \( C \) not on \( \overrightarrow{AB} \) (use point tool)

4. Construct segment \( \overline{AC} \) and \( \overline{BC} \) (use segment tool)

5. Place point \( D \) on \( \overrightarrow{AB} \), and outside the triangle. (use pt. on object tool)

6. Find the measure of \( \angle BCA \), \( \angle BAC \), and \( \angle DBC \). (use angle in the measurement tool)

What connection do you see between the three angle measurements?

________________________________________________________________________
________________________________________________________________________
________________________________________________________________________

7. Calculate the measure \( \angle BCA + \angle BAC \).\(^1\) (Calculator Tool)

What do you notice about your calculation vs. your angle measurements?

________________________________________________________________________

\(^1\) Select the tabulate tool from the measurement toolbox. (Ninth button). To define the size and location of your table, move the pointer to an unoccupied location in the drawing window. Press and hold the mouse button down. Move the cursor to another location to draw a rectangle. Release the mouse button. A table appears within the rectangle. Resize the table to 4 columns and 10 rows, by dragging the lower right corner
8. Draw a table with 4 columns and 10 rows. \(^2\) *(Tabulate Tool)*

9. Move your pointer to the measure of \(\angle BCA\). Click on this measurement. You should see the measurement in the first column. Repeat this for \(\angle BAC, \angle DBC\), and the Result measurement.

<table>
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<tr>
<th></th>
<th>(BCA)</th>
<th>(BAC)</th>
<th>(DBC)</th>
<th>Result</th>
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<tr>
<td>1</td>
<td>47.35</td>
<td>60.77</td>
<td>108.13</td>
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10. Use animation to find data. \(^3\)

Examine the table. What do you notice? Is your conclusion from number 7 consistent?

11. From the construction above, you should have noticed that the sum of the remote angles equals the exterior angle for any triangle. This is called the Exterior-Angle Theorem.

\[ \angle 1 + \angle 2 = \angle 4 \quad \text{(Exterior-Angle Theorem)} \]

\(^2\) Select the tabulate tool from the measurement toolbox. (Ninth button). To define the size and location of your table, move the pointer to an unoccupied location in the drawing window. Press and hold the mouse button down. Move the cursor to another location to draw a rectangle. Release the mouse button. A table appears within the rectangle. Resize the table to 4 columns and 10 rows, by dragging the lower right corner.

\(^3\) Move your mouse to the display toolbox. Select animation from the selection of tools. Next click on your table, then go to either point A or point C and hold the left button down as you move your mouse away from the point. A spring should appear as you do this. When you release the left button, the point you chose should move, and the table should fill up. Click the mouse to stop the process.
12. Use your conclusion to solve for the variable x.

13. What is the relationship between $\angle 3$ and $\angle 4$?

14. How does this relationship explain the exterior-angle theorem?

GOAL II: Students will be able to explain the relationship between supplementary angles and the exterior angle theorem.

Application Problem
15. You are in charge of landscaping at a local business. You spent all morning pouring concrete for a sidewalk. Now it is time to water the new grass that is bordered by the new sidewalk and a triangular flowerbed. (See picture below) There is a programmable sprinkler head that needs to be set for the number of degrees to be watered. You can not measure angle 3, because of the new cement and grass. Find angle 4 by the provided information.
Diagram of a sprinkler system with labeled parts:

- Flowerbed
- New grass
- Sprinkler head
- Wet cement

Measurements:

- \( c_1 = 42.7 \)
- \( c_2 = 36.2 \)
Lesson I: Triangles- Exterior Angle Theorem- Solutions

GOAL 1: Students will be able to find the measure of an exterior angle from the two remote interior angles of a triangle.

INVESTIGATE USING CABRI GEOMETRY II

6. Find the measure of $\angle BCA$, $\angle BAC$, and $\angle DBC$. (use angle in the measurement tool)
   Check Students Work
   What connection do you see between the three angle measurements? Angle $DAB = \angle CAB + \angle ACB$. Some students may not see the connection until later in the activity.

7. What do you notice about your calculation vs. your angle measurements? The sum of two remote angles equals the measure of the exterior angle

9. The students value may be different depending on their diagram.

10. Examine the table. Is your conclusion from number 7 consistent?
   At this time, teachers should have a discussion of student results. Students should have the sum of two remote angles equals the measure of the exterior angle. When they move one of the remote vertexes, the conclusion should be consistent.

GOAL II: Students will be able to explain the relationship between supplementary angles and the exterior angle theorem.

12. Use your calculations to solve for the variable x.
   For the first diagram, $x = 35$. For the second diagram, $x = 70$.

13. What is the relationship between Angle 3 and Angle 4?
   The angles are supplementary.

14. How does this relationship explain the exterior-angle theorem?
   Since angle 3 + angle 4 = 180, and angle 1 + angle 2 + angle 3 = 180, angle 4 = angle 1 + angle 2.

Application Problem
15. Find angle 4 by the provided information.
   Since we can not measure angle 3, because of the cement and grass, we can use the exterior-angle theorem. Angle 1= 42.7, and angle 2= 36.2. The sum of angle 1 + angle 2 =78.9. So angle 4 equals 78.9.