Discovering Euler’s Theorem

Lesson Summary:
The students will be placed in pairs to help them learn to cooperate and help one another through self-discovery and the cooperative activity. The students will work individually on journal prompts. This will help develop creativity and written communication skills.

Key Words:
Euler, prisms, edges, faces, sides

Prior Knowledge:
- Students will have prior lessons on polygons, polyhedrons.
- Students will have prior experience in recognizing patterns.
- Students must have some prior geometry and problem solving skills.

This information helps the teacher to plan the daily lesson prior to this activity to ensure that students have this prior knowledge. It will be important to know that the students can recognize patterns.

Objectives:
Students will discover and analyze the relationship between the number of faces, sides, and edges in various geometric configurations.
- Self-discovery will help the students become self-learners.
- Students will collaborate with others.

Materials:
- Overhead to give students information
- Computers/Calculators equipped with Cabri Geometry II.
- Lab Handout
- Journal Prompt

Suggested Procedure:
1. Introduction
   A. Quick Set Induction to gain student interest.

1. Group activity
   A. Students will get into pairs.
   B. The pairs group will work together to construct various polygons on Cabri.
   C. Using the constructions, the pairs will complete a table and answer various questions.
   D. The teacher will monitor the groups to ensure all group members are participating and to assist with any questions

1. Conclusion
   A. Class discussion of today’s discoveries.
B. Class summary and journal assignment will conclude the lesson.

This lesson is designed for a Math class consisting of 10th and 11th grade students in Geometry. The time allocated will be one day with a take home assignment.

**Grouping:**
Students will be put in pairs for the self-discovery and cooperative activity. They will take notes and observe individually during whole group instruction. Students will also have an individual take home journal prompt.

**Students with Special Needs:**
Special Needs students will be grouped with others and will be able to participate in group activities and peer collaboration. Available equipment will be used if students are visually or hearing impaired. Low-level students may be given modified questions to answer in addition to group activity to access their level of understanding if different than other group members.

**Assessment:**
- Possible points for class and group participation will be 10. (This will take the place of a quiz score). Teacher will observe students to ensure that each student participates and assists other group members.
- Possible points for journal prompt will be 10 also.
- Each student will be given a review handout in two days that will also cover this material and it will be worth 30 points.
Euler’s Theorem

Team Members’ Names: ___________________________________________________

File name: ________________________________________________________________

Goal 1: Analyze the relationship between the number of faces, sides and edges in various geometric configurations.

Investigate Using Cabri Geometry II*

1. Construct a pyramid.
   a. First choose the show axes tool.
      Then use the define grid tool to display grid points.

   b. Construct a plane polygon in the first quadrant. [use regular polygon tool]

   c. Construct a point not in the plane polygon. [use point tool]

   d. Connect each vertex of the plane polygon to this point. [use segment tool]
      (If you grab a vertex, you can modify the shape and appearance your pyramid.)

2. What kind of polyhedron is a pyramid? Why?

________________________________________________________________________
________________________________________________________________________
3. Use your construction of the pyramid to complete the second row of the following table.

   Given that a polygon has \( n \) sides, express the number of vertices, edges, and faces using \( n \).

<table>
<thead>
<tr>
<th>Figure</th>
<th>Vertices</th>
<th>Edges</th>
<th>Faces</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polygon</td>
<td>( n )</td>
<td>( n )</td>
<td>( n )</td>
</tr>
<tr>
<td>Pyramid</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Double Pyramid</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cube</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cylindrical Solid</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4. Now construct a double pyramid.
   Follow the same procedure as the pyramid, except you will create two additional points not in the plane polygon.
   The two points should be separated by the plane polygon.

5. What kind of polyhedron is a double pyramid? Why?
   ________________________________________________________________
   ________________________________________________________________

6. Use your observations of the double pyramid to complete the third row of the table.

7. Construct a cube.
   a. Construct a plane polygon.
   [use regular polygon tool]
   b. Construct a second, equivalent plane polygon above the first.
   c. Connect the vertices of the first plane polygon with the corresponding
   [use segment tool]
vertices of the second.

8. Use your observations of the cube to complete the fourth row of the table.

9. Construct a cylindrical solid.
   a. Construct a pentagon.
      [use regular polygon tool]
   b. Construct a second, equivalent pentagon above the first.

   d. Connect the vertices of the first pentagon, with the corresponding vertices of the second.
      [use segment tool]

9. What do you observe about the cube and the cylindrical solid? Why is this so?

______________________________________________________________________
______________________________________________________________________

10. Use your observations of the cylindrical solid to complete the fifth row of the table.
Your table should now be complete!

11. Do you notice a pattern? Explain.

_________________________________________________________________________

12. Using your observations, create a generalization that relates the number of faces, edges, and vertices in a polyhedron?

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