Conic Sections and Locii

Lesson Summary:
Students will investigate the ellipse and the hyperbola as a locus of points. Activity One addresses the ellipse and the hyperbola is covered in lesson two.

Key Words:
Locus, ellipse, hyperbola

Background knowledge:
familiarity with Cabri program;
knowledge of the definition of locus, ellipse, and hyperbola

NCTM Standards addressed:
Analyze characteristics and properties of 2 dimensional geometric objects; select and use different representational systems, including coordinate geometry and graph theory; recognize the usefulness of transformations and symmetry in analyzing mathematical situations; use visualization and spatial reasoning to solve problems both within and outside mathematics.

Learning objectives:
The students will construct an ellipse as a locus of points.
The students will construct an hyperbola as a locus of points.

Materials needed:
Cabri program

Suggested procedure:
Split students into groups of two or three and have them complete the worksheet.
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Activity Two: The Ellipse as a Locus

Group Member’s Names: ____________________________
File name: _______________________________________

Goal: Construct an ellipse and explore it as being a locus.

1. Open a new figure and display the rectangular coordinate system. [Show Axes tool]

2. Draw a horizontal segment \( \overline{AC} \) across the top of the screen. [Segment tool]

3. Draw a point \( B \) on segment \( \overline{AC} \). [Point tool]

4. Draw point \( F_1 \) on the positive x-axis and reflect this point across the y-axis to point \( F_2 \). [Reflection tool]

5. Use the Compass tool to draw a circle with radius equal to the distance \( AB \) centered at point \( F_1 \).

6. Repeat this procedure to draw a circle with radius equal to the distance \( BC \) centered at point \( F_2 \). Use the Intersection Point tool to construct the points of intersection of the two circles.

7. Your figure should look like this:

8. Animate point \( B \) on segment \( \overline{AC} \). [Animate tool]

9. What happens to the radius of the circles as \( B \) moves along the segment? What
happens to the points of intersection of the circles as B moves along the segment? Make a conjecture about the path of these points of intersection as B moves.

10. Construct the locus of the points of intersection of the circles as B moves on segment $\overline{AC}$. You will need to execute the Locus command twice, once for each intersection point. What does this locus create?

11. Use the Conic tool to draw a conic on top of the two loci that were just created. Select three of the points defining the conic near the y-axis on one locus, and the other two points near the y-axis of the other locus. Do not use the intersection points of the circles to define the conic.

12. Draw point P anywhere on the conic. Construct segments $\overline{PF_1}$ and $\overline{PF_2}$ using the Segment tool.

13. Use the Distance & Length tool to find the lengths of segments $\overline{PF_1}$ and $\overline{PF_2}$. Label each distance accordingly. Also, use the Calculate tool to find the sum of the distances of PF1 and PF2. Label this sum as $PF_1 + PF_2$.

14. Animate point P using the Animate tool, and examine the values that were just calculated. What happens to the distances of PF1 and PF2? What happens to the sum of these distances as P moves? Using this knowledge, define an ellipse as a locus.
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Activity Two: The Hyperbola as a Locus

Group Member’s Names: ___________________________________________________________

File name: ___________________________________________________________________

Goal: Construct a hyperbola and explore it as being a locus.

1. Open a new figure and display the rectangular coordinate system. [Show Axes tool]

2. Draw a horizontal segment across the top of the screen. [Segment tool]

3. Draw points A, B, and C on this segment. [Point tool]

4. Draw point F1 on the positive x-axis and reflect this point across the y-axis to point F2. [Reflection tool]

5. Use the Compass tool to draw a circle with radius equal to the distance AB centered at point F1.

6. Repeat this procedure to draw a circle with radius equal to the distance AC centered at point F2. Use the Intersection Point tool to construct the points of intersection of the two circles.

7. Your figure should look like this:
8. Animate point $A$ on the segment. [Animate tool]

9. What happens to the radius of the circles as $A$ moves along the segment? What happens to the points of intersection of the circles as $A$ moves along the segment? Make a conjecture about the path of these points of intersection as $A$ moves.

10. Construct the locus of the points of intersection of the circles as $A$ moves on segment $\overline{AC}$. You will need to execute the Locus command twice, once for each intersection point. What does this locus create?

11. Use the Conic tool to draw a conic on top of the two loci that were just created. Select three of the points defining the conic on one locus, and the other two points on the other locus. Do not use the intersection points of the circles to define the conic.

12. Draw point $P$ anywhere on the conic. Construct segments $PF_1$ and $PF_2$ using the Segment tool.

13. Use the Distance & Length tool to find the lengths of segments $PF_1$ and $PF_2$. Label each distance accordingly. Also, use the Calculate tool to find the difference of the distances of $PF_1$ and $PF_2$. Label this result as $PF_1 - PF_2$.

14. Animate point $P$ using the Animate tool, and examine the values that were just calculated. What happens to the distances of $PF_1$ and $PF_2$? What happens to the difference of these distances as $P$ moves? Using this knowledge, define a hyperbola as a locus.